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# Tropical Cyclones and Polar Lows: Velocity, Size, and Energy Scales, and Relation to the 26°C Cyclone Origin Criteria

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**ABSTRACT**

The goal of this paper is to quantitatively formulate some necessary conditions for the development of intense atmospheric vortices. Specifically, these criteria are discussed for tropical cyclones (TC) and polar lows (PL) by using bulk formulas for fluxes of momentum, sensible heating, and latent heating between the ocean and the atmosphere. The velocity scale is used in two forms: (1) as expressed through the buoyancy flux $b$ and the Coriolis parameter $l_{\rm c}$ for rotating fluids convection, and (2) as expressed with the cube of velocity times the drag coefficient through the formula for total kinetic energy dissipation in the atmospheric boundary layer. In the quasistationary case the dissipation equals the generation of the energy. In both cases the velocity scale can be expressed through temperature and humidity differences between the ocean and the atmosphere in terms of the reduced gravity, and both forms produce quite comparable velocity scales. Using parameters $b$ and $l_c$, we can form scales of the area and, by adding the mass of a unit air column, a scale of the total kinetic energy as well. These scales nicely explain the much smaller size of a PL, as compared to a TC, and the total kinetic energy of a TC is of the order $10^{18} - 10^{19}$ J. It will be shown that wind of 33 m s$^{-1}$ is produced when the total enthalpy fluxes between the ocean and the atmosphere are about 700 W m$^{-2}$ for a TC and 1700 W m$^{-2}$ for a PL, in association with the much larger role of the latent heat in the first case and the stricter geostrophic constraints and larger static stability in the second case. This replaces the mystical role of 26°C as a criterion for TC origin.

The buoyancy flux, a product of the reduced gravity and the wind speed, together with the atmospheric static stability, determines the rate of the penetrating convection. It is known from the observations that the formation time for a PL reaching an altitude of 5–6 km can be only a few hours, and a day, or even half a day, for a TC reaching 15–18 km. These two facts allow us to construct curves on the plane of $T_{\rm s}$ and $\Delta T = T_{\rm s} - T_a$ to determine possibilities for forming an intense vortex. Here, $T_a$ is the atmospheric temperature at the height $z = 10$ m. A PL should have $\Delta T > 20$°C in accordance with the observations and numerical simulations. The conditions for a TC are not so straightforward but our diagram shows that the temperature difference of a few degrees, or possibly even a fraction of a degree, might be sufficient for TC development for a range of static stabilities and development times.

**Key words**: typhoons, polar lows, velocity, size and energy scales, cyclone origin criteria

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## 1. Introduction

Intense atmospheric vortices (IAV) are generated because of substantial thermodynamic disequilibrium between the ocean surface water and the atmosphere (Riehl, 1950; Kleinschmidt, 1951), but special conditions are required for their actual formation (Gray, 1968; Lystad, 1986). The goal of this paper is to develop simple analytical means to describe some basic features and evolution stages of intense atmospheric vortices, namely, tropical cyclones (TC) and polar hurricanes, or polar mesocyclones, sometimes also known

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as polar lows (PL). This approach explains their sizes, total kinetic energy, the initial stage of the development, and their subsequent quasistationary stage. Some IAV steady state features have been described in a short article (Golitsyn, 1997) where the scales for TC and PL sizes, winds, and kinetic energy (Golitsyn, 1980; Boubnov and Golitsyn, 1990, 1995) have been calculated on the basis of the theory of convection in rotating fluids. In the decade since that article was published, work on destructive hurricanes has focused on their birth, development, and evolution even more. The evidently developing feature of global warming makes the question of what could be expected in a warmer climate even more important.

Emanuel (2003) showed in a recent review that the problems of TC origin and the initial stage of development were less studied and understood. Modern weather forecast models can describe the evolution of existing TCs, and even of much smaller PLs, with some accuracy. Efimov et al. (2007) described the evolution of a quasi-TC formed at the end of September 2005 over the western part of the Black Sea by using the MM5 (Fifth-generation Penn State University Mesoscale Model). However, they used a horizontal resolution of 10 km and had initial data fields with the same resolution. These data fields revealed a weak cyclonic vortex in the middle troposphere over the area of the sea. If this vortex had been absent in the initial data, the model would not have shown the origin and the development of a hurricane-type cyclone. It should be noted that the value of the SST ($T_{\rm s}$) was about 22°C and the temperature of the air mass coming from the Balkan Peninsula, was 10°C–12°C colder. However, Efimov et al. (2007) described a hindcasting case. The problem for operationally forecast TCs and PLs remains unsolved.

Many researchers have attempted to develop vortex or TC genesis parameters (e.g., DeMaria et al., 2001; Pelevin and Rostovtseva, 2004). These parameters include the vertical and horizontal wind shear, atmospheric stability, and midtropospheric moisture (DeMaria et al., 2001), which best explained the instability of the vortex sheet along the shear line for the climatologically oceanic regions where TCs are predominantly found. NASA recently organized a special observational field program in the Atlantic region in an attempt to understand TC generation (Kumar, 2006).

Gaertner et al. (2007) recently developed statistics for quasi-tropical cyclones over the western Mediterranean by using an ensemble of seven regional models with a resolution of 50 km and 30 years simulation for periods of 1961–1990 and 2071–2100. Boundary conditions from the Hadley Center model (HadCM) have been used for the end of this century using the moderate IPCC (Intergovernmental Panel on Climate Change) SRES-A2 scenario for the greenhouse gas evolution. The observed SST have been used for the first period. It was found that the number of quasi-TCs was, on average, increasing together with their lifetimes, though the resolution was rather inadequate for strong vortices, and further work is needed. One of the study results indicates that the problem could be serious for Southwestern Europe in a warmer climate.

There is a large amount of evidence (e.g., Lystad, 1986; Emanuel and Rotunno, 1989) that large developing IAVs, TCs, and PLs have specific hydrodynamic structures when thermodynamical contrasts between the ocean and the atmosphere are strong enough. The science community is used to the idea that Benard cells and rolls are typical convective hydrodynamic structures. TCs and PLs are specific hydrodynamic structures on a much larger scale, as well. The most well known condition for the origin of TCs, established 60 years ago by Palmen (1948), is that $T_{\rm s}$ should be equal to or larger than 26°C, and has to include not only $T_{\rm s}$, but the temperature and the humidity of the atmosphere, as well. Then, it is possible to estimate the enthalpy fluxes from the ocean to the atmosphere feeding the IAVs for hurricane winds of 33 m s$^{-1}$. These fluxes differ between TCs and PLs. Their values and the differences will be estimated and explained in section 3 based on the so-called bulk formulas in section 2 (e.g., Kraus and Businger, 1994) in conjunction with two types of the wind velocity parameterizations. The first case uses a velocity scale from the theory of convection in rotating fluids. The second case exploits the parameterization of the kinetic energy dissipation rate in the boundary layer above the sea surface, as proposed by Bister and Emanuel (1998) (see also Emanuel, 2003). Both cases produce similar wind estimates, though both describe the steady state of IAVs. For lack of any better representation, these two parameterizations will be further used as functions of the thermodynamical contrast between the ocean and the atmosphere to estimate IAV development rates.

The observations reveal that full development can extend up to 17–18 km in height, i.e., up to the tropical tropopause, and within in about one day, or even in half a day (Palmen and Newton, 1969). Polar hurricanes reach their upper boundary of 5 to 6 km much faster, in only a few hours (Rasmussen and Turner, 2003). The short time for IAV development presents an important constraint for models of any complexity.

The only substantial mechanism of enthalpy transfer from the ocean to the atmosphere is convection. It is determined by the temperature difference between the ocean and the atmosphere $\Delta T = T_{\rm s} - T_{\rm a} > 0$. This difference changes during the day. Due to large thermal inertia, the oceanic surface temperature $T_s$ varies little, while the atmospheric temperature $T_a$ decreases at night because of thermal radiation escaping to space. Therefore, the convection over the ocean is mostly developed during the dawn hours, which is demonstrated by the presence of clouds. One can judge how high the convection propagates into the atmosphere from the clouds' upper boundary. This height is regulated by the buoyancy input into the atmosphere and by its static stability, to be further discussed in section 4. Climatologically, the atmosphere above the boundary layer is always stable, and the colder the underlying surface is, the larger the static stability parameter is (Mokhov, 1994; Mokhov and Akperov, 2006). The convection with these conditions is called "penetrative" and there are simple, very important, and successful estimates and diagnoses of its development rate (Zubov, 1945; Turner, 1973; Zilitinkevich, 1987, 1991) as a function of the buoyancy flux, time, and atmospheric stability. These estimates, together with the constraint of sufficiently fast IAV development, allow us to find the ranges of $T_s, \Delta T$, and atmospheric humidity. These results are discussed in section 5.

This approach may be also used to describe the formation of hot cloud towers (HT). These objects are formed in tropical depressions (TD) caused by other tropical disturbances, mostly by easterly waves (Riehl, 1954). Only about one tenth of TDs can develop into TCs. HT merging is thought to be the usual method by which TCs are formed (e.g., see Montgomery et al., 2006 and references therein), but it might be not the only way.

Section 6 discusses theoretically the merging of linear and extended vortices, described in 2D with conditions for such process occurrences, and with interpretations of the processes' physical meaning. The HT merging idea is also mentioned by Emanuel (2003). He also notes that there might be several ways for large IAVs to form. At the end, we urge for a detailed analysis of the satellite data, preferably of the geostationary kind, related to the formation of TCs and PLs. A thorough analysis of the time slices of weather and climate models reproducing early stages of IAV formation is of a great importance for the development of operational criteria, as well. Such criteria would be also useful to understand tendencies for the birth of IAVs, and for their behavior in a warmer climate.

## 2. The air-sea interaction and the total buoyancy flux

### 2.1 *Momentum, heat, and moisture fluxes*

For the reader's convenience, here we briefly present the derivation of the total buoyancy flux between two media. This derivation is a basic element throughout the subsequent material. It includes both sensible and latent heat. Their sum will be called the enthalpy, after Emanuel (1991), because it is the enthalpic change of sea surface water providing its heat as sensible heating and evaporation for the atmosphere.

The sea surface temperature $T_s$ is assumed here to be the cold film temperature. At enthalpy fluxes, $f$, of the order 1 kW m$^{-2}$ it will be about half a degree colder than the bulk temperature of the ocean's upper mixed layer. The temperature difference within the cold film is proportional to $f^{3/4}$ (Kraus and Businger, 1994; Ginzburg et al., 1977, etc.). These effects should be taken into account when considering the effect on the thermal regime of the ocean, though the cold film at high wind speeds and waves exists only in some parts of the ocean surface.

The fluxes of momentum, sensible, and latent heat at the separation boundary between the water and the air are described by the so-called bulk formulae (Kitaigorodskii, 1973; Smith, 1989; Kraus and Businger, 1994; Fairall et al., 2003, etc.):

$$\tau = \rho\langle w'u'\rangle = c_D\rho U^2 = \rho u_*^2 \ , \quad (1)$$

$$F_{SH} = \rho c_p\langle w'T'\rangle = c_T\rho c_p U\Delta T \ , \quad (2)$$

$$F_{LH} = \rho\langle q'w'\rangle = c_E\rho U\Delta q \ , \quad (3)$$

where $\rho$ is the air density, $U$ is the wind velocity at 10 m, the reference height above the mean water surface, $w'$ and $u'$ are the fluctuations of the vertical and horizontal wind components, $c_D, c_T, c_E$, are the drag coefficients for the momentum, sensible heat, and moisture, $\Delta T = T_s - T_a$ is the temperature difference between the sea surface and the air at 10 m, $\Delta q$ is the difference between the mixing ratios of the water vapor at the sea surface where $T = T_s$ and the relative humidity is 100%, and the one at $T$ ($z = 10$ m). The angular brackets denote statistical averaging.

According to ocean climatology (Fleming, 1942), the relative air humidity $r = q(T)/q_s(T)$ over all oceans at all latitudes in all seasons is close to 80%, i.e., $r \approx 0.8$. It decreases to 0.7 only occasionally. Emanuel (1991) in his review uses $r = 0.75$. In this case, the Bowen number, the ratio of the sensible and latent heat fluxes, would be smaller than here by a factor of 25/20=1.25. Later on we shall use $r = 0.8$, which may sometimes decrease the water vapor flux considerably.

The mixing ratio $q$ is related to the specific humidity, $e(T)$ is water vapor pressure:

$$q = \frac{\mu_w}{\mu_a}\frac{e}{p} = 0.622\frac{e}{p} \ , \quad (4)$$

where $\mu_{\rm w} = 18.015$ and $\mu_{\rm a} = 28.97$ are the molecular weights of the vapor and the air, $p$ is atmospheric pressure. Both gases are considered here as ideal ones.

The formulae (1)–(3) follow from similarity considerations and are obtained by the analogy with a technical problem of a plate cooled by a colder air flow with velocity $U$. The coefficient $c_{\rm T}$ in such a problem is known as the Stanton number (e.g., Monin and Yaglom, 1971).

### 2.2 *Buoyancy flux*

Because the molecular weight of the vapor is less than that of the air, the air's humidification increases its buoyancy relative to dry air. The density of an air parcel with saturated water vapor is equal to

$$\begin{aligned}\rho &= \rho_{\rm a} + \rho_{\rm w} \\ &= \frac{p_{\rm a}}{R'_{\rm a}T} + \frac{e_{\rm s}}{R'_{\rm w}T} \\ &= \frac{p_{\rm a}}{R'_{\rm a}T}\left(1 + \frac{\mu_{\rm w}}{\mu_{\rm a}}\frac{e_{\rm s}}{p_{\rm a}}\right) \\ &\cong \frac{p_{\rm a}}{R'_{\rm a}T}\left(1 + 0.622\frac{e_{\rm s}}{p}\right) ,\end{aligned} \tag{5}$$

where $p = p_{\rm a} + p_{\rm w}$ is the total gas pressure, and $R'_{\rm a}$ and $R'_{\rm w}$ are the gas constants for dry air and water vapor. The decrease of the air density, as compared with the dry air at the same pressure, is equal to

$$\begin{cases} \Delta\rho = -\dfrac{0.622}{R'_{\rm a}T}p\Delta q = -\dfrac{0.378 e_{\rm s}}{R'_{\rm a}T} , \\ 0.378 = \dfrac{\mu_{\rm a} - \mu_{\rm w}}{\mu_{\rm a}} . \end{cases} \tag{6}$$

As a result, the total buoyancy flux due to the air heating and moisturizing in the Boussinesq approximation is found to be equal to

$$b = -\frac{g}{\rho}\langle \rho' w' \rangle = -\frac{g}{\rho T}\langle w'T' \rangle - 0.622 g \langle w'q' \rangle .$$

Using the bulk formulae (2) and (3), we obtain from here

$$b = -c_{\rm T}\frac{\Delta T}{T} g U \left(1 + 0.622\frac{c_{\rm E}}{c_{\rm T}} T \frac{\Delta q}{\Delta T}\right) . \tag{7}$$

As measurements show, the drag coefficients $c_{\rm E}$ and $c_{\rm T}$ do not differ from each other (Smith, 1989; Fairall et al., 2003). We can re-write Eq. (7) in a more practically convenient form while taking account of Eqs. (4) and (6) as

$$\begin{cases} b = -g'U , \\ g' = c_{\rm T}\dfrac{\Delta T}{T} g \left(1 + 0.378 T \dfrac{\Delta e}{p\Delta T}\right) , \\ \Delta e = e_{\rm s}(T_{\rm s}) - r e_{\rm s}(T_{\rm a}) , \end{cases} \tag{8}$$

where the value $g'$ may be called the reduced gravity (acceleration). Following Henderson-Sellers (1984), the saturated water vapor pressure as a function of temperature can be conveniently presented (units: Pa) as

$$e_{\rm s}(T) = \exp[23.7812 - 4157(T - 33.91)^{-1}] , \tag{9}$$

$$e_{\rm a}(T) = \exp[23.8014 - 4157(T - 33.91)^{-1}] . \tag{10}$$

These expressions were obtained from the integration of the Clapeyron-Clausius equation by some approximations and are valid with the accuracy of a few hundredths of a percent in all practical ranges of atmospheric temperatures. There are other formulae for the air humidity, but we use the ones above in the calculations below. The difference in the first term in the square brackets in Eqs. (9) and (10) is related to the fact that for water with salinity 34 promille the pressure of the saturated water vapor is 98% of the pressure over distilled water (Kraus and Businger, 1994).

The first formula (8) shows that the buoyancy flux is the product of the reduced gravity acceleration and the wind velocity. It has dimensions of power per unit mass. In statistically stationary conditions, i.e., in a steady state, this flux is, evidently, equal to the rate of generation and dissipation of the convective kinetic energy (per unit mass). This accords with the convective energy balance equation (see e.g., Monin and Yaglom, 1971). This relationship does not hold in nonstationary conditions, e.g., the buoyancy flux in penetrative convection (see below section 4) decreases linearly with height, expending its energy to warm the air (Turner, 1973; Zilitinkevich, 1987, 1991).

Equation (8) determines the buoyancy flux over the water surface in the absence of water vapor condensation. Observations (and calculations) show that for TC, as well as for PL, the condensation starts at levels of a few hundred meters (300–400 m) above the ocean surface. In TC the clouds rise up to the tropical tropopause, i.e., up to 15–18 km. In PL the cloud tops reach 5–6 km (Rasmussen and Turner, 2003). In this case, there should not be a substantial error if we neglect the relatively thin subcloud layer and consider that the latent heat starts to be felt right at the water surface. In any case, the enthalpy flux leaving the water is just the sum of sensible heat and latent heat.

We can now rewrite Eq. (8) as

$$\begin{cases} b = -c_{\mathrm{T}}\dfrac{\Delta T}{T}g(1+\mathrm{Bo}^{-1})U = -g'U\,, \\ g' = c_{\mathrm{T}}g\dfrac{\Delta T}{T}(1+\mathrm{Bo}^{-1})\,, \end{cases} \quad (11)$$

where we introduce the Bowen number as the ratio of the sensible and latent heat fluxes. The inverse ratio is equal to

$$\mathrm{Bo}^{-1} = \frac{\mu_{\mathrm{w}}}{\mu_{\mathrm{a}}}\cdot\frac{L\Delta e}{pc_p\Delta T} = 0.610L_1\frac{\Delta e}{\Delta T}\,. \quad (12)$$

Here $L = 10^6 L_1$ is the latent heat of evaporation, and $p = p_{\mathrm{s}} = 1.013\cdot 10^5$ Pa is the air pressure at the sea level. According to Henderson-Sellers (1984), the latent heat in SI units is a function of the absolute temperature in the following form

$$L = 1.91846\times 10^6[T(T-33.91)^{-1}]^2\,. \quad (13)$$

In the observed range of temperatures of the ocean's open surface from $-2$°C to $+31$°C, the value $L_1 = 10^{-6}L$ changes from 2.506 to 2.430, i.e., by 3%. This should be taken into account for exact calculations.

Note that Eqs. (7) and (8) show that the buoyancy flux exists even at $\Delta T = 0$, i.e., when $T_{\mathrm{s}} = T_{\mathrm{a}}$ is satisfied. Then the flux is only due to the lighter water vapor as compared to the unsaturated air. The last flux vanishes when the air is warmer than the water, as it follows from Eq. (8):

$$\Delta T = T_{\mathrm{a}} - T_{\mathrm{s}} = 0.378\Delta e/p\,. \quad (14)$$

For the Tropics at $T = 300$ K, or 27°C, and $\Delta e/p \approx 10^{-2}, \Delta T \approx 1$ K is produced. The molecular diffusion flux is too small to be taken into account. The value of the reduced gravity in the subcloud layer is related to the difference in the molecular weights of the water and the air. The appearance of the condensation heat substantially increases the buoyancy flux. The ratio of the second term in the square brackets of Eq. (8) to the similar term in Eq. (11) is equal to $m = 0.608c_pT/L$. As the temperature increases from 271 K to 300 K, its value changes from 6.5% to 7.5% because of a decrease of the evaporation heat due to increase of the temperature, as Eq. (13) shows.

The fluxes of the sensible and latent heat, and therefore the buoyancy flux, depend on the wind speed $U$ see Eqs. (1)–(3) and (7) or (11). The convective vortices are processing the ambient air by gathering it from large distances and by raising it up. These processes create high winds by concentrating the ambient angular momentum, which intensifies the convective vortices, increasing the fluxes in its turn (Emanuel, 1991, 2003). These are highly non-linear processes, but fortunately the situation can be described by invoking wind parameterization depending on fluxes. We will do so by using two methods. One method will use the results of the Golitsyn (1980) and Boubnov and Golitsyn (1990, 1995) based on theoretical results for convection in rotating fluids. The other method will use the expression for the kinetic energy dissipation in the atmospheric boundary layer proposed by Bister and Emanuel (1998) (and see also Emanuel, 2003). Both parameterizations for the buoyancy flux and velocities presented here produce comparable results.

## 3. Scaling for steady vortices

### 3.1 *Convection in rotating fluids*

Strictly speaking, theoretical, experimental, and numerical results in this part of the study have been obtained for stationary conditions, i.e., for many revolutions. On average the lifespan of a TC is about a week (Riehl, 1954; Golitsyn et al., 1999). A PL can exist for a few days or less (Mokhov and Priputnev, 1999). Therefore, the theory should produce reliable results for mature vortices in respect to their size, winds, and total kinetic energy. One can hope that it gives the results of the right order of magnitude even for the vortex development stage. Later we will see that the air in arising vortices rotates about 20 times faster than the Earth's rotation rate (see section 6). In any case, we cannot suggest anything better.

The dimensional parameters determining the convection are: the buoyancy flux $b$, the Coriolis parameter $l_{\mathrm{c}}$, and the geometric scale $d \ll L$. We can form the following non-dimensional number from these values:

$$Ro = b^{1/2}l_{\mathrm{c}}^{-3/2}L^{-1}\,, \quad (15)$$

which has been found to be the Rossby number (Golitsyn, 1980). The rotation's strong influence is felt for motions with $Ro \ll 1$. Further on, we assume that the characteristic motion scales are much less than the geometric scale $d \ll L$. We can define the area and velocity scales from $b$ and $l_{\mathrm{c}}$:

$$S = bl_{\mathrm{c}}^{-3} \approx d^2\,, \quad (16)$$

$$U = cb^{1/2}l_{\mathrm{c}}^{-1/2}\,. \quad (17)$$

After adding the weight of the atmospheric column unit $M_1 = 10^4$ kg m$^{-2}$ to the determining parameters, we can write the kinetic energy scale for the vortex as:

$$E = \frac{1}{2}M_1SU^2 = \frac{c^2}{2}M_1\frac{b^2}{l_{\mathrm{c}}^4}\,. \quad (18)$$

The value of the numerical coefficient $c$ in the velocity scale (17) has been measured by many researchers. Boubnov and Golitsyn (1990, 1995) found it to be $1.7 \pm 0.1$; Fernando et al. (1991) gave $c = 1.7 \pm 0.14$; Maxworthy and Narimosa (1994) presented $c = 1.8 \pm 0.15$, and they also confirmed the scaling (16). We will use $c^2 = 3 \cong 1.73^2$ in this paper later on. We should mention that the scale for the size where $d = \varepsilon^{1/2} l_c^{-3/2} = S^{1/2}$, where $\varepsilon$ is the rate of the kinetic energy dissipation per unit mass, first proposed by Hopfinger et al. (1982) to describe the results of their experiments on turbulence generated by an oscillating grid in a rotating vessel.

Recalling the Bernoulli equation we can define the pressure drop scale as:

$$\Delta p = \rho U \approx \rho b l_c^{-1} \,.$$

This relationship was tested by Golitsyn et al. (1999). Using data for about 2000 TC, it was found that there is indeed a close correlation between the central pressure drop and the value of $\rho U^2$, but with a numerical coefficient about 1.5. Our formula gives $\Delta p \approx 32$ hPa for $\rho = 1.2$ kg m$^{-3}$, $b = 3 \times 10^{-2}$ m$^2$ s$^{-3}$, and $l_c = 0.5 \times 10^{-4}$ s$^{-1}$ and with numerical coefficients 1.5 and 3 for $U^2$.

We shall estimate in section 4 that the buoyancy flux for TCs is of the order of $3 \times 10^{-2}$ m$^2$ s$^{-3}$, and it could be twice that for PLs. For the tropics at latitude $\theta = 20°$, we have $l_c = 0.5 \times 10^{-4}$ s$^{-1}$ and for polar regions at $\theta = 70°$, it is $1.37 \times 10^{-4}$ s$^{-1}$. Equation (18) produces $3 \times 10^{18}$ J for the scale of the kinetic energy of the TC (see details of the buoyancy flux evaluation in section 4). This answers James Lighthill's question (see e.g., Lighthill et al., 1994) as to why the energy of a TC can reach $10^{18} - 10^{19}$ J. Using the size scale $D = S^{1/2}$ from Eq. (16) we can estimate that for the same values of the flux $b$ the size of a TC should be 4.5 larger than for a PL. But taking into account that the buoyancy flux for a PL should be about twice or more of that for a TC (see Tables 3 and 4), we find that the ratio of the diameters should decrease to $4.5/2^{1/2} \approx 3$ Thus, PL sizes should be on average only a third of TC sizes, as it is observed in nature (Rasmussen and Turner, 2003).

After combining Eqs. (8), (11), and (16) we can obtain an expression for wind velocity through reduced gravity:

$$U \approx 3 g' l_c^{-1} \,, \;\; g' = c_T \frac{\Delta T}{T} g (1 + \mathrm{Bo}^{-1}) \,, \qquad (19)$$

or as a function of the sea-air temperature difference $\Delta T$, temperature $T_s$, and the air humidity. For $T_s = 300$ K or 27°C, $\Delta T = 1$ K, the relative air humidity $r = 0.8$, and $l_c = 0.5 \times 10^{-4}$ s$^{-1}$ (at latitude $\theta = 20°$), and we obtain $U = 34$ m s$^{-1}$. To get $U \geqslant 33$ m s$^{-1}$ for polar mesocyclones at $\theta = 70°$, $T_s = 275$ K, we need $\Delta T \geqslant 22$ K. This result matches well the numerical findings (see Rasmussen and Turner, 2003) that the sea-air temperature difference should be no less than 20 K in order to get a PL to form (a PL may have wind velocity less than 33 m s$^{-1}$).

An important consequence of our approach is an obvious statement that the ocean surface must be warmer than air, at least, for a TC (the author could find such a statement only in one popular book, see Moline, 1964). This explains why TCs are not observed in the spring or in the early summer when the atmosphere is usually warmer than the ocean.

A TC cools the sea surface temperature by extracting the enthalpy from the ocean upper mixed layer. Zhu and Zhang (2006) found that the SST cooling by a typhoon causes the weakening of the central pressure drop by about 20 hPa after a 1 K cooling. A similar magnitude of 18–20 hPa deepening was found for a 1 K warming of SST. These authors also stressed an importance of having a wind-driven sea wave model coupled with the typhoon and ocean models in order to have a better description of the momentum and enthalpy fluxes.

### 3.2 *Wind parameterization by Bister and Emanuel* (*1998*)

This paper showed that the main kinetic energy dissipation for a TC takes place within the boundary layer over the ocean, $h_1 \sim$1–2 km thick. This dissipation does not explicitly depend upon the thickness $h_1$, and is equal to

$$D = c_D \rho U^3 \,, \qquad (20)$$

where $c_D$ is the drag coefficient, which is in itself a function of the wind. Following Emanuel (2003), for the time being, we will not take into account this dependence, assuming $c_D = 2 \times 10^{-3}$. According to the data treated by Fairall et al. (2003), $c_D$ increases from $1.3 \times 10^{-3}$ to about $2.3 \times 10^{-3}$ as the wind ranges from 10 to 25 m s$^{-1}$. Then, according to Powell et al. (2003) and Donelan et al. (2004), it reaches about $2.5 \times 10^{-3}$ for winds of about 30 m s$^{-1}$, and after that it starts to decrease.

We estimate the mean dissipation of the kinetic energy per unit mass from Eq. (20) as:

$$\varepsilon = \frac{D}{\rho h_1} = \frac{c_D U^3}{h_1} \,. \qquad (21)$$

The kinetic energy generation at steady state equals the dissipation, wherefrom the mean energy generation equals the buoyancy flux $b$. Then, combining Eqs. (8), (11), (19), and (20), one obtains:

$$U = \left(\frac{g' h_1}{c_D}\right)^{1/2} , \quad g' = c_T \frac{\Delta T}{T} g(1 + \mathrm{Bo}^{-1}) . \quad (22)$$

At typical TC values of $g' = 7 \times 10^{-4}$ m s$^{-2}$ (see Table 2 below), $h_1 = 2$ km, and $c_D = 2 \times 10^{-3}$, we find $U \approx 28$ m s$^{-1}$. Due to the large uncertainties in $h_1$ and $c_D$ values, one may agree that the parametrizations give close results.

### 3.3 *Atmospheric thermodynamics, buoyancy and heat fluxes, and wind in dependence on thermodynamic disequilibrium*

We assume in this subsection that the ocean surface water, as well as the near surface air does not significantly change temperature, at least for the time of an IAV formation. This is not a very obvious assumption because, as we will see in section 5, the full developmental span of an IAV could be a day or so. It is known that TCs can cool the surface waters by 1°C to 4°C (Emanuel, 1991, 2003). But IAVs are moving structures and, if so, they would encounter fresh unperturbed water while moving. The requirement then is that $(\partial T_S/\partial x)d \approx \Delta T$, i.e., the temperature difference on the scale of the TC diameter $d$ should be considerably less than the acting temperature difference $\Delta T = T_s - T_a$. A similar constraint could be imposed on the time changes: $(\partial T_S/\partial t)\tau \approx \Delta T$, where $\tau$ is the time scale for air parcel turnover in the convective process. This is the Deardorff scale for steady convection: $\tau = h/w_* \approx h^{2/3} b^{-1/3}$. For $h = 15 \times 10^3$ m and $b = 3 \times 10^{-2}$ m$^2$ s$^{-3}$ we have $\tau \approx 2 \times 10^3$ s$\approx 33$ min. One can safely assume that the ocean temperature would not change significantly over half an hour.

Table 1–4 present all the necessary numbers both for PLs and TCs showing us a picture of the required thermodynamic disequilibrium quantitatively.

The next tables present values of the buoyancy flux, the enthalpy flux leaving surface waters with temperature , and the wind velocity scale according to Eqs. (17) and (19) for polar and tropical latitudes.

We see from Table 3 that the hurricane winds of 33 m s$^{-1}$ require enthalpy fluxes of about 1700 W m$^{-2}$. Such measured values have been reported at high latitudes over open waters in winter near Newfoundland and east of Japan (Yu. A. Volkov, S. S. Lappo, both private communications in the early 1990s).

We see from Table 4 that in the Tropics the hurricane winds reach 33 m s$^{-1}$ as the enthalpy fluxes are at about 700 W m$^{-2}$. It is twice less than for PLs. As we will see later in section 6, it is related to the much stronger static stability of the atmosphere at high latitudes compared with the TC cases and also to the geostrophic constraints, which are much more strict at high latitudes than at low ones. A crucial role is also played by the latent heat fluxes, which are relatively small at high latitudes but play a chief role for the Tropics.

Other results for different $T_s$ and $\Delta T$ values can be readily obtained by extrapolation. In other such cases, $T_s$ and $\Delta T$ could be easily calculated from this section's equations.

**Table 1.** The reduced gravity and 1+Bo$^{-1}$ value for polar regions at $\Delta T = 20$ K and $l_c$ at 70°N, $r = 0.8$, according to Eqs. (11) and (12).

| $T_s$ (°C) | $10^3 g'$ (m s$^{-2}$) | 1+Bo$^{-1}$ |
|---|---|---|
| 0 | 1.32 | 1.39 |
| 2 | 1.36 | 1.44 |
| 4 | 1.41 | 1.50 |
| 6 | 1.46 | 1.57 |
| 8 | 1.53 | 1.66 |

## 4. Atmospheric stratification and penetrative convection

The main enigma with TCs, as stated by Emanuel (1991), is why they are relatively rare, with only about 80 per year on the global scale, mostly occurring in the Northern Hemisphere where areas of the ocean surface with $T_s \geqslant 26$°C are vast and can feed TCs in considerably larger numbers. All seven conditions by Gray (1979) play a role here. From his viewpoint the prime and the most important condition is the static stability of the atmosphere. It is determined by the vertical density gradient, which is derived from the adiabatic gradient:

$$\Gamma = \left(\frac{d\rho}{dz}\right)_a - \frac{d\rho}{dz} , \left(\frac{d\rho}{dz}\right)_a = -\frac{\rho g}{c_a^2} , \; c_a^2 = \chi R'_a T , \quad (23)$$

where $c_a$ is the adiabatic sound velocity in an ideal gas, and $\chi = c_p/c_v$ is the adiabatic exponent. The hydrostatic equation $dp = -\rho g dz$ is used here as well. A convenient measure of the density vertical gradient is the square of the Brunt-Väsälä frequency $N^2 = -(g/\rho)\Gamma$, which in the Boussinesq approximation is equal to

$$N^2 = -\frac{g}{T}\left(\frac{dT}{dz} - \gamma_a\right) , \quad \gamma_a = -\frac{g}{c_p} = -9.8 \quad (24)$$

If a source of buoyancy at the lower boundary of the stably stratified fluid starts to act, then a convective boundary layer develops and its height increases with time $t$ as (see Zubov, 1945; Turner, 1973; Zilitinkevich, 1987, 1991)

$$h(t) = N^{-1}(2bt)^{1/2} . \quad (25)$$

**Table 2.** The reduced gravity and 1+Bo$^{-1}$ value for the Tropics at 20°N.

| $T_s$ (°C) | $10^4 g'$ (m s$^{-2}$) | | 1+Bo$^{-1}$ | |
|---|---|---|---|---|
| | $\Delta T = 2$ K | $\Delta T = 1$ K | $\Delta T = 2$ K | $\Delta T = 1$ K |
| 24 | 5.23 | 3.90 | 6.04 | 10.1 |
| 26 | 5.87 | 4.91 | 6.75 | 11.3 |
| 27 | 6.20 | 5.20 | 7.15 | 12.0 |
| 28 | 6.52 | 5.49 | 7.55 | 12.7 |
| 29 | 6.87 | 5.77 | 7.98 | 13.4 |
| 30 | 7.20 | 6.05 | 8.43 | 14.1 |

**Table 3.** The buoyancy, enthalpy fluxes, and wind for the polar regions at $\Delta T = 20$ K, $\theta = 70$°N, $r = 0.8$, according to Eqs. (11), (2), (3), (8), and (19).

| $T_s$ (°C) | $100b$ (m s$^{-2}$) | $F$ (W m$^{-2}$) | $U$ (m s$^{-1}$) |
|---|---|---|---|
| 0 | 3.81 | 1310 | 29 |
| 2 | 4.05 | 1404 | 30 |
| 4 | 4.37 | 1511 | 31 |
| 6 | 4.67 | 1633 | 32 |
| 8 | 5.13 | 1796 | 33.5 |

This expression does not take account of the process of entrainment of the ambient air, which increases by some 20 percent the r.h.s. of the Eq. (25) [see a detailed analysis by Zilitinkevich (1987, 1991)]. A typical value of the atmospheric temperature vertical gradient $\gamma = dT/dz$ is about $-6$ K km$^{-1}$ or even larger, and it is about $-6.5$ K km$^{-1}$ at low latitudes (Mokhov, 1994; Mokhov and Akperov, 2006). The latter value yields $N \approx 1 \times 10^{-2}$ s$^{-1}$, and for the high latitudes, where $\gamma = 5.5$ K km$^{-1}$, $N \approx 1.24 \times 10^{-2}$ s$^{-1}$.

We know from Gray (1979) that TCs are developed in near neutral stability conditions. As we have already seen above, the time for the development would be less than an hour in a purely neutral case, which is certainly not the real case. So, in order to quantitatively appreciate conditions that work, we have to analyze Eq. (25) in detail.

## 5. A criterion for formation of an intense atmospheric vortex

Here we shall use the time constraint that an IAV should develop in a stable atmosphere in half a day ($4 \times 10^4$ s≈ 11 h) for TCs and in half that time in PLs. We rewrite Eq. (25) as an inequality

$$\frac{(2bt)^{1/2}}{hN} \geqslant 1 \tag{26}$$

with a condition that time $t$ would be more than enough to reach the altitude $h$. We present Eq. (26) as a condition for the development, or in a more detailed form with the help of Eqs. (11) and (19) as

$$\begin{cases} \dfrac{c_{\mathrm{T}} g}{hN}\left(\dfrac{2t}{l_{\mathrm{c}}}\right)^{1/2} \dfrac{\Delta T}{T}(1+\mathrm{Bo}^{-1}) = A\dfrac{\Delta T}{T}(1+\mathrm{Bo}^{-1}) \geqslant 1\,, \\ A = \dfrac{c_{\mathrm{T}} g}{hN}\left(\dfrac{2t}{l_{\mathrm{c}}}\right)^{1/2}\,, \end{cases} \tag{27}$$

where we denote $A$ as a multiplier, not dependent on thermodynamic disequilibrium conditions. At typical values of $b \approx 3 \times 10^{-2}$ m$^2$ s$^{-3}$ and $N = 1 \times 10^{-2}$ s$^{-1}$ the time for the vertical development of convection to height $h$=18 km would be several days. This clearly illustrates the importance of small static stability. We take the case of $N = 3 \times 10^{-3}$ s$^{-1}$, which corresponds to a difference of only about 0.3 K km$^{-1}$ between the actual and the dry adiabatic gradients. Figure 1 presents the criterion (27) on the plane $(\Delta T, T_s)$. The curve with the numbers larger or equal to 1 covers the area of the parameter space with a possibility of a TC development at 20°N in 11 h ($4\times10^4$ s). Due to the independence of multiplier $A$ in Eq. (27) and either $\Delta T$ or $T_s$ this graph has a wider interpretation. For instance, the curve labeled 0.8 can become the curve labeled 1 on the same plane if we increase $A$ by a factor 1.25, e.g., by increasing time by a factor of $1.25^2 = 1.5625$, or decreasing the Brunt-Väisälä frequency by a factor of 1.25. A similar interpretation could be applied to the curves with numbers larger than 1, by decreasing time or increasing the Brunt-Väisälä frequency. Figures 1a and 1b differ only in the values of the relative humidity of the atmosphere at 10 m. Replacing $r$=0.8 in Fig. 1a with a value of 0.7 in Fig. 1b increases the inverse Bowen ratio by a factor of 1.5. Only this fact can decrease the temperature difference $\Delta T$ down to a fraction of a degree, still giving the possibility for TC formation at high SST.

In Fig. 2 we present similar results for the case of polar lows, or PLs, at $\theta = 70$°N, $N = 1.0 \times 10^{-2}$ s$^{-1}$ and with $t = 2 \times 10^4$ s or 5.5 h. The graph is in agreement with the numerical resultss requiring a large temperature difference ($\Delta T \geqslant 20$ K) in order to form a PL. The same interpretation for similar corre-

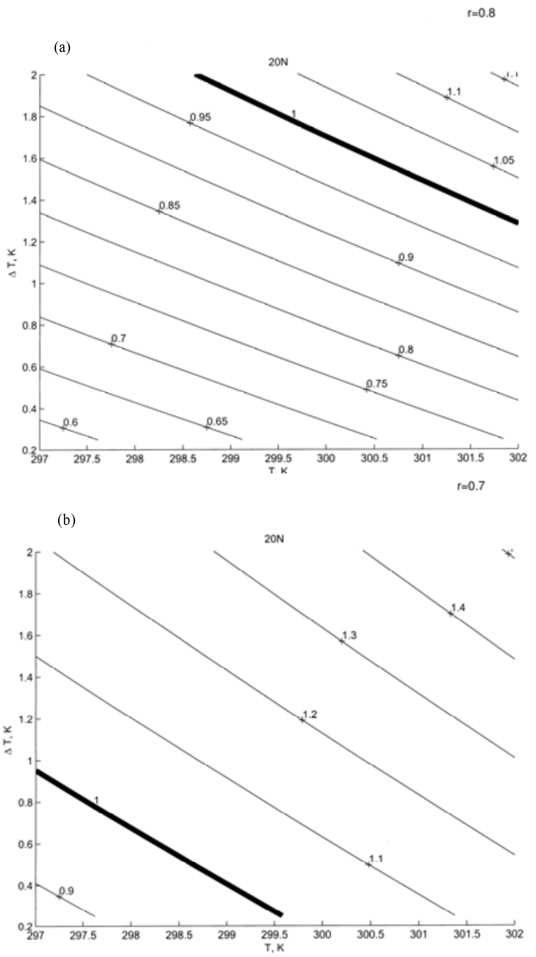


**Fig. 1.** Diagram for tropical cyclone origin conditions depending on sea surface temperature $T_s$ and the temperature difference between the ocean $T_s$ and the atmosphere $T_a$ at $z = 10$ m. (a) for the relative humidity $r = 0.8$ and (b) for $r = 0.7$. The thick line labeled 1 is the critical curve where development of a tropical cyclone becomes possible, for parameter space to the right and above this curve.

sponding curves can be given in case of a TC, due to the independence of the multiplier $A$ on the thermodynamic disequilibrium parameters.

In order to investigate the sensibility of the criterion (26) to different parameterizations of the velocity and buoyancy flux, we use the Bister and Emanuel (1998) formula for this flux as in Eqs. (11) and (22) for Eq. (26). In order to have a common multiplier $(\Delta T/T_s)(1 + Bo^{-1})$ we take square of Eq. (26). Then as in Eq. (27) we will have a multiplier $A_b$ not dependent on the thermodynamic parameter. The ratio of the two multipliers is:

$$R = \frac{A}{A_b} = \left(\frac{c_D h N}{h_b}\right)^{1/3} (2t)^{-1/6} l_c^{-1/2} . \quad (28)$$

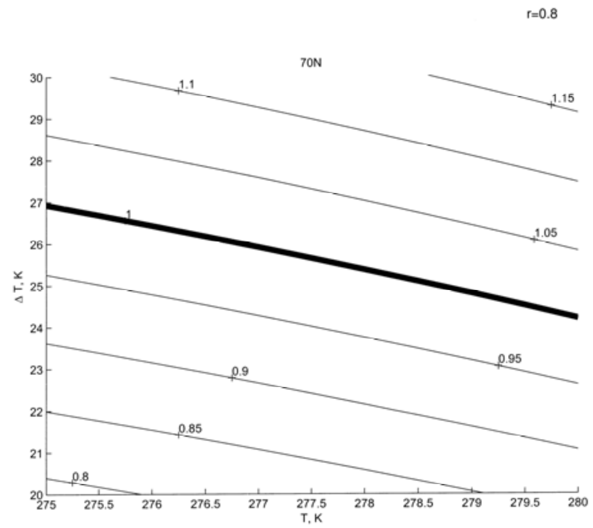


**Fig. 2.** Same as in Fig. 1, but for the origin of a polar low (PL), at $r = 0.8$.

We have values of this ratio as $R = 1.10$ for TC and $R = 0.78$ for PL for the times $t$ used above, and for the conditions adopted to calculate the two preceding graphs. Due to the crude assumptions used in the derivations of our criteria, one may say that both forms are close to each other.

## 6. Problems for intense atmospheric vortex development

### 6.1 *Spatial and temporal constraints on IAV origin*

In reality, for this approach to the formation of IAVs, the potential area for their genesis should be large enough or at least greater than the scale of a vortex as given by Eq. (16), wherefrom the diameter $d \approx S^{1/2} = b^{1/2} l_c^{-3/2}$. The homogeneity of the water surface temperature leads to a natural condition:

$$\frac{dT_s}{dx} d \ll \Delta T = T_s - T_a , \quad (29)$$

where $x$ is the horizontal coordinate.

Gray (1979) presents the small size of the vertical wind shear as an important condition for TC formation, which is obviously a condition for PLs as well. This condition could be written as

$$t_{dvp} \Delta U \ll d , \quad (30)$$

where $\Delta U = (dU/dz)h$ is the ambient wind velocity difference between the lower and upper parts of a developing vortex, $t_{dvp}$ is its development time, and condition (30) is assuring together with (29) that the vortex would have enough energy to feed its development and not be sheared apart.

**Table 4.** Same as in Table 3 for tropics, $\theta = 20°$N.

| $T_s$ (°C) | $100b$ ($m^2\ s^{-2}$) | | $F_s$ (W $m^{-2}$) | | $U$ (m $s^{-1}$) | |
|---|---|---|---|---|---|---|
| | $\Delta T = 2$ K | $\Delta T = 1$ K | $\Delta T = 2$ K | $\Delta T = 1$ K | $\Delta T = 2$ K | $\Delta T = 1$ K |
| 24 | 1.86 | 1.46 | 6770 | 528 | 32.8 | 28.0 |
| 26 | 2.03 | 1.61 | 733 | 582 | 34.2 | 30.5 |
| 27 | 2.12 | 1.70 | 768 | 613 | 35.0 | 31.4 |
| 28 | 2.22 | 1.78 | 803 | 643 | 35.8 | 32.1 |
| 29 | 2.33 | 1.87 | 841 | 674 | 36.7 | 32.9 |
| 30 | 2.44 | 1.97 | 879 | 705 | 37.6 | 33.8 |

In section 5 we analyzed crucial effects of the atmospheric static stability. Of course, it was only a crude approximation. The most important problem here is to specify the influence of the condensation heat, which is supplied for the whole volume of a cloud and is dependent on height and time. An analytical approach here is highly desirable, especially for TCs, where the conditions for their origin are much more subtle than for PLs, though one would hope that the results presented here would not change drastically because they agree reasonably with what we know about these vortices from observations.

Figure 1 demonstrates how constrained the conditions are for the inequalities (26) or (27) to take place. If we assume a TC's developmental time is one day, then according to our discussion, the value of $\Delta T = T_s - T_a$ should be $\sqrt{2} \approx 1.4$ times less, which poses a severe requirement on the precision with which these temperature values or their differences have to be measured.

Now we want to comment about some aspects of TC origin. Riehl (1954) noted a high correlation with easterly waves, especially for Atlantic TCs. The easterly waves have wavelengths of 1500–2000 km, and pressure and temperature amplitudes of order 1 hPa and 1 K, respectively. The value of $\Delta T$ in the negative temperature deviation phase is increasing, and this may explain the aforementioned correlation, because Fig. 1 demonstrates how important one degree Kelvin of temperature difference is for the convection, or perhaps even less differential is required at the warmest end of possible SST extremes.

The convection cycle over the ocean in the Tropics is reverse in its diurnal course compared to over land, where there is a maximum in the afternoon, while the maximum intensity of over the ocean occurs near dawn. This is because the ocean has a large thermal capacity and its surface temperature is nearly constant while the atmosphere at night is cooled by thermal radiation into space. Therefore, the night seems to be a preferable time for the formation of TCs. Infrared (IR) multichannel observations from geostationary satellites should be analyzed in detail, especially at night, with high time resolution, in order to try to track TC formation.

Emanuel (2003) noted that there could be several TC origin mechanisms. Recent numerical schemes with high spatial and temporal resolution propose growth and subsequent merging of hot towers (HT) cloud systems with heights up to 18 km and diameters of 20–30 km (Hendriks et al., 2004; Montgomery et al., 2006, and see the literature cited therein). HTs develop in tropical depressions (TD), which have about 20 to 30 HTs. Only about one tenth of TDs can develop into TCs. TDs, which develop into a TC, have about 30 to 40 HTs at early stages.

In the extensive series of computations by Hendriks et al. (2004) and Montgomery et al. (2006) it was demonstrated how HTs develop, acquire initial vorticity, and merge, forming a TC and intensifying the total vorticity by bringing in the planetary momentum of the ambient air from thousands of kilometers away. A thorough analysis of the process in time evolution is highly desirable, in order to follow the process of TC formation, at least numerically.

### 6.2 *Merging of hot towers as a TC formation mechanism*

The merging of HTs leads to the appearance of an intense hurricane vortex. It forms from a tropical depression with $N = 30$ to 40 HTs. These objects have initial vorticity, and merging intensifies the total vorticity of a single large vortex. For a circular vortex its intensity is measured by the velocity circulation:

$$\Gamma = 2\pi r u = S\Omega = \pi r^2 \Omega \,, \tag{31}$$

where $r$ is the vortex radius, $S$ is its area, and the vorticity is:

$$\Omega = \frac{2u}{r} \,, \tag{32}$$

(see Kochin et al., 1964; Saffman, 1992). We want to estimate what values of vorticity (and velocity) could be associated with a single cloud tower. This may be done in the following way. We can deduce from the vorticity conservation theorem in a non-viscous flow that the final vorticity in the process of evolution should be of the same order as, or may be less than, that the sum of the initial cloud towers vorticities. The final vorticity of a developed hurricane can be estimated as $\Omega_{\rm f} = 2u_{\max}/r_{\max}$, where $r_{\max}$ is the radius of maximal winds $u_{\max}$. Thus we could write:

$$\Omega_{\rm f} = 2u_{\max}/r_{\max} \leqslant Nu_t/r_t \ . \tag{33}$$

With $u_{\max} \approx 40$ m s$^{-1}$, $r_{\max} = 50$ km, $N = 40$, and $r_t = 10$ km for an HT we can obtain an estimate of its tangential velocity $u_t \leqslant (2u_{\max}r_t)/(Nr_{\max})$ from Eq. (33). With these numbers we obtain a reasonable estimate $u_t \leqslant 0.5$ m s$^{-1}$ for a HT, quite a modest value. At least from this point of view this merging process could be a feasible mechanism for typhoon formation.

Studying the process of vortex merging started from the theoretical works by Aref (1979) and Novikov (1976). They first described the merging in a three vortex configuration of different signs. Then Novikov (1976, 1980) developed an approach to study the problem of an arbitrary number of vortices in 2D, both linear and spatial ones (see Novikov and Sedov, 1979, 1983; Dobritsyn and Sedov, 1987; Sedov, 1995). Calculations in 2D could be considered for the barotropic atmosphere, which may describe the neutrally stratified case. There are conditions in all studied cases when the major vortex parts could concentrate in space and merge, while increasing the initial vorticity.

This bears a resemblance to the inverse energy cascade both in two dimensions (Kraichnan, 1967; Batchelor, 1969) and in the quasigeostrophic approximation (Charney, 1971). In both cases the energy from smaller perturbations flows to the larger ones, forming larger structures in the course of the time. Such processes have been modeled in numerical computations and in laboratory experiments (Paret et al., 1999).

### 6.3 *TCs' role in the general atmospheric circulation*

Tropical cyclones play a substantial role in the maintenance of the general atmospheric circulation in the Northern Hemisphere (NH). In the second half of the year, the TC season, they transport into the middle latitudes about half of the moisture and angular momentum in their latitudinal belt (Palmen and Newton, 1969, and see also Golitsyn, 1997). Whether their role changes and how in a warmer climate is an important practical question.

There is also one more interesting feature related to TCs that has not been mentioned before, to the author's knowledge. In the convective process the vertical heat flux is associated with the mass flux. Of excourse, the total flux averaged over a large area is zero, but this way the convection is circulating large air volume, or mass. The density of the mass flux is equal to

$$m = \langle \rho' w' \rangle = \rho b/g \ , \tag{34}$$

when taking account of Eqs. (2) and (7). This simple expression, which is not widely known, was used firstly by Monin (1970), and then by Golitsyn (1979). We can crudely estimate how much air TCs are processing through themselves in a year's time. With Eq. (34), the area scale (6) mean lifetime of a TC equal to $6 \times 10^5$ s, or one week, for latitude 20°N, and for a typical value of the buoyancy flux $b = 3 \times 10^{-2}$ m$^2$ s$^{-3}$ from our Table 2, we obtain $10^{18}$ kg. The total mass of our atmosphere is $5 \times 10^{18}$ kg. If we take into account that most TCs are in the NH, one may say that about a third of the NH atmosphere is processed by TCs during their season.

### 6.4 *TCs in a warmer climate, and requirements for instrumentation precision*

Subsections 6.1–6.3 show that vortex merging has been already known for about 30 years in theory, laboratory studies, and numerical calculations. But it was realized only recently that this merging could happen at much larger atmospheric scales. The problem is now to visualize the process from geostationary satellite data and from computations and to compare these to each other. This study may, hopefully, serve as a guide for such an analysis by specifying what quantities should be analyzed and with which accuracy. As we saw, the accuracy should be quite high and might be on the threshold of the precision of the state of the art of present instrumentation and data retrieval. Numerical results may offer greater hope in this respect.

As we stated in the introduction, the goal of such studies should be a development of operational criteria for determining locations of potential TC development. As a byproduct, these results could serve as a base from which to develop an outlook for TC statistics and intensity in a warmer climate of the future. The climatology of such operational conditions would be of high practical importance and give a basis for any such outlook. The author dares also to think that this exercise might serve an educational purpose, as well.

Numerous discussions are happening now concerning TC intensities and statistics in a warmer climate, and concerning whether the increase of the global temperature during the past thirty years has revealed itself in this respect. The discussion has been started by Emanuel (2005) and Webster et al. (2005), and prolonged by many others. Probably the most extended analysis for 1986–2005 belongs to Klotzbach (2006). He noted that during these two decades the

SST warming was in the range 0.2°C–0.4°C for major ocean basins. The data shows a large increase in TC intensity and lifetime for the North Atlantic and a decrease in the Northeast Pacific. Other basins do not reveal any significant trends in numbers, nor in intensity. These results confirm that besides SST there are several other important factors governing intensity and frequency of TCs, as stated by Gray (1968, 1979). At the same time, Trenberth and Shea (2006) analyzed TC activity for the North Atlantic in 2005 and concluded that a part of the increase in activity in 2004 and 2005 could be due to the higher than normal sea surface temperature.

Our study proposes that the temperature difference between SST and atmosphere $\Delta T = T_s - T_a$, together with water vapor pressure $\Delta e = e(T_s) - re_s(T_a)$ is among the major factors controlling vortex sustainability. Due to the large ocean thermal inertia, its transient response is much slower than that of the atmosphere. Due to that effect, the atmosphere warms up a bit faster, therefore decreasing $\Delta T$ despite that fact that the larger warm pool area may decrease the TC number. In any case, this problem requires very thorough checking. A decrease of $\Delta T$ could be an important negative feedback on the number and intensity of IAVs. For a example, this may be because high latitudes are warming faster and more than the middle latitudes, as observed in the Arctic, and thus one may expect fewer numbers of PLs and their intensity could be smaller with climate change.

The effects for TCs could be of the order of a few tenths of a degree, which are found to be important enough to distinguish between cases that generate cyclones or fail to generate them (see Fig. 1). This poses severe requirements for the precision (and the stability) of instruments measuring SST, tropospheric temperature (and humidity), vertical profiles, etc. Figure 1 clearly illustrates that the higher the SST, the smaller the temperature difference needs to be to cause TC development, again due to the Clausius-Clapeyron equation.

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## APPENDIX

### Similarity Consideration

One may ask why the results of a convective theory for rotating fluids tested in a laboratory could be used for TCs and PLs. The main dynamic similarity criterion in the atmosphere for a rotating planet is the Rossby number:

$$Ro = U/l_c L \,, \tag{A.1}$$

where $L$ is a linear external scale of the flow. In an earlier work for convection in a rotating fluid (Golitsyn, 1980), the Rossby number ($Ro$) was first assumed to be $\ll 1$, and then the velocity scale (17) was derived. The linear scale $L_i = S^{1/2} = (b/l_c^3)^{1/2}$ was first proposed by Hopfinger et al. (1982). If $L_e$ and $U$ from Eq. (17) are put into Eq. (15), $Ro = 1$ can be obtained. Laboratory studies listed above show that scales $L_i = S^{1/2}$ and Eq. (17) start to appear even at $Ro$=O(1).

Observations and numerical results show that the external spatial scale involved in the formation of a vortex, i.e., the space from which the air converges into it, is much larger than the internal scale $L_i$. For instance, the scales for formation in "A most beautiful polar low" (paper by Nordening and Rassmussen, 1992) are several thousand kilometers, as revealed by spiral cloud streets around that PL. For $U \sim 30$ m s$^{-1}$, $l_c = 1.4 \times 10^{-4}$(70°N) and $L = 2 \times 10^6$ m we get $Ro \approx 0.1$. For a TC with $U \sim 40$ m s$^{-1}$, $l_c = 0.5 \times 10^{-4}$ (20°N), and the value of $L = 3000$ km we obtain Ro$\approx 0.25$. The fact that our scales nicely explain features for both IAV types confirms the applicability of our reasoning.

These intense vortices arise due to the thermodynamic disequilibrium between the atmosphere and the ocean. This unstable situation is resolved by convection forming an ascending body of air. This rise causes air to converge into the plume, coming from large distances, a mechanism which brings into the vortex ambient angular momentum. This intensifies the vortex and its azimuthal winds, which extract more and more enthalpy from the upper ocean. The process is leveled off when the dissipation of the kinetic energy within the atmospheric boundary layer equals its rate of generation in the convection process (Emanuel, 1991, 2003).

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