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A Comparison Study of the Methods of Conditional Nonlinear Optimal Perturbations and Singular Vectors in Ensemble Prediction

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ABSTRACT

The authors apply the technique of conditional nonlinear optimal perturbations (CNOPs) as a means of providing initial perturbations for ensemble forecasting by using a barotropic quasi-geostrophic (QG) model in a perfect-model scenario. Ensemble forecasts for the medium range (14 days) are made from the initial states perturbed by CNOPs and singular vectors (SVs). 13 different cases have been chosen when analysis error is a kind of fast growing error. Our experiments show that the introduction of CNOP provides better forecast skill than the SV method. Moreover, the spread-skill relationship reveals that the ensemble samples in which the first SV is replaced by CNOP appear superior to those obtained by SVs from day 6 to day 14. Rank diagrams are adopted to compare the new method with the SV approach. The results illustrate that the introduction of CNOP has higher reliability for medium-range ensemble forecasts.

Key words: ensemble prediction, medium-range forecasts, forecast skill, spread, Talagrand diagram

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1. Introduction

Medium-range ensemble forecasts are currently a part of the operational activities of many major numerical weather prediction centers, such as the European Centre for Medium-Range Weather Forecasts (ECMWF) and the National Centers for Environmental Prediction (NCEP) (Mureau et al., 1993; Molteni et al., 1996; Toth and Kalnay, 1993, 1997; Tracton and Kalnay, 1993). This important development is the results of two decades of theoretical research and numerical experimentation following the work of Epstein (1969).

Leith (1974) opened the door for ensemble forecasting, by presenting a practical way to predict the evolution of the probability distribution function of atmospheric states. Much of the utility of Monte Carlo techniques is based on its interpretation as a random sample of a probability distribution. However, the enormous number of variables in modern global atmospheric prediction models has led to the general conclusion that it is impossible to sample adequately the full complexity of the prediction model's phase space. Instead, an attempt is made to identify a vastly reduced set of directions in phase space that is believed *a priori* to be of most importance to sample well, and these directions are frequently limited by dynamical constraints.

At ECMWF, the SV approach proved to be successful for generating initial condition perturbations for ensemble forecasting (Mureau et al., 1993; Molteni et al., 1996). This method was first considered in a meteorological context by Lorenz (1965) in a predictability study with a low-dimensional barotropic model. SVs are defined in such a way as to maximize the growth of perturbation total energy in the early

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part of the forecast, which provides the main rationale for their use in the context of ensemble prediction (Mureau et al., 1993). Ehrendorfer and Tribbia (1997) demonstrated that forecast-error covariance for a specified lead time can be predicted most efficiently using an ensemble constructed in the subspace of the leading analysis-error covariance SVs under assumptions of linearity of error growth and normality of errors, which presented the theoretical justification for the use of SVs in ensemble prediction systems. For medium-range forecasts, total-energy SVs are probably a reasonable substitute for analysis-error covariance SVs (Molteni et al., 1996). Gelaro et al. (2002) computed the SVs with an analysis error variance metric, which showed that the leading SV are consistent with the expected distribution of analysis errors. Hamill et al. (2003) calculated the flow-dependent analysis-error covariance SVs and further suggested that operational ensemble forecasts based on total energy SVs could be improved by changing the type of SVs used to generate initial perturbations. Leutbecher (2005) studied the impact on the ECMWF ensemble prediction system of using SVs computed from 12-h forecasts instead of analyses. His results showed that the computation of SVs from forecasts could be used to disseminate the ensemble forecasts earlier or to allocate more resources to the nonlinear forecasts. From the above references, it seems that refining the use of SVs has become one of the trends for development of ensemble predictions.

However, Anderson (1997) pointed out that using singular vector decomposition (SVD) to determine the directions in which the evolution would be most sensitive is only relevant in a linear regime, and fails to give information about the likelihood of the extreme perturbations. Gilmour and Smith (1997) also reported that there are limits based on linearity assumptions in the construction of ensemble perturbations. Given that SVs cannot capture the nonlinear characteristics, Mu et al. (2003) proposed a new concept called CNOPs, which is a natural extension of SVs into the nonlinear regime. Based on the successful application of SVs at ECMWF to ensemble forecasting, Mu and Jiang (MJ, 2008) have attempted to use CNOPs as initial perturbations for ensemble prediction. The conclusions were that the use of CNOP improves the ensemble mean forecast skill in the medium range when the analysis error is a kind of fast growing error. The present study is an extension of that companion paper. Moreover, limited experiments were illustrated in previous studies, and only the ensemble mean was adopted as the evaluation criterion. In this paper, 13 different cases have been considered, assuming that analysis errors have almost the same magnitude as the perturbation vectors. In addition, various evaluation methods are adopted, which allows considerably greater confidence in the results presented.

Several forecast experiments in the presence of a theoretically perfect model are designed in the next section. In section 3, the relationships between the forecast skill and the spread are illustrated. Section 4 further gives evaluation of reliability for the above numerical experiments by using Talagrand diagrams. Conclusions and some discussion follow in section 5.

2. The experimental design

All experiments here are conducted in a perfect-model framework with the barotropic QG model used in MJ (2008). This is a double periodic channel model. The domain is $2X \times 2Y$, specifically 6400×3200 km^2, with 32(16) grid points in zonal (meridional) direction. The model performs 144 time steps per day.

In this research, we choose three different true initial states shown in Fig. 1, which are given respectively by the streamfunctions:

$$\psi_{\mathrm{tr},01} = 0.5\sin\left(\frac{2\pi x}{2X}\right) + \sin\left(\frac{2\pi y}{2Y}\right) + 0.5\ , \qquad (1)$$

$$\psi_{\mathrm{tr},02} = -0.5\sin\left(\frac{2\pi x}{2X}\right) + \sin\left(\frac{2\pi y}{2Y}\right) - 0.5\ , \qquad (2)$$

$$\psi_{\mathrm{tr},03} = \sin\left(\frac{4\pi x}{2X}\right) + \cos\left(\frac{2\pi y}{2Y}\right) - 0.8\ . \qquad (3)$$

Subsequently, the true state of the system can be found by running the nonlinear model from the above specific given initial condition. In the real world, a true state like this can never be known because of inevitable observation errors. We assumed that "observed" data can be generated for each true point by adding a random selection from a prescribed normal distribution. The observational field can be controlled by the adjustment of the standard deviation and random seed. All experiments employ a simple four-dimensional variation data assimilation scheme, described in detail in MJ (2008), for acquisition of the control analyses. The control forecasts are then produced with the same model, starting from the control analyses. The observational data distribution is specified so that the error of the control analysis, which is the control analysis minus the true initial state, is about 9‰ from the true initial state, as measured by the root mean squared (rms) error. The magnitude of analysis error broadly agrees with that of current analysis systems. For the first true initial state, eight different observational data distributions lead to eight cases with different control analysis fields; for the second true initial state, two cases; and for the third true

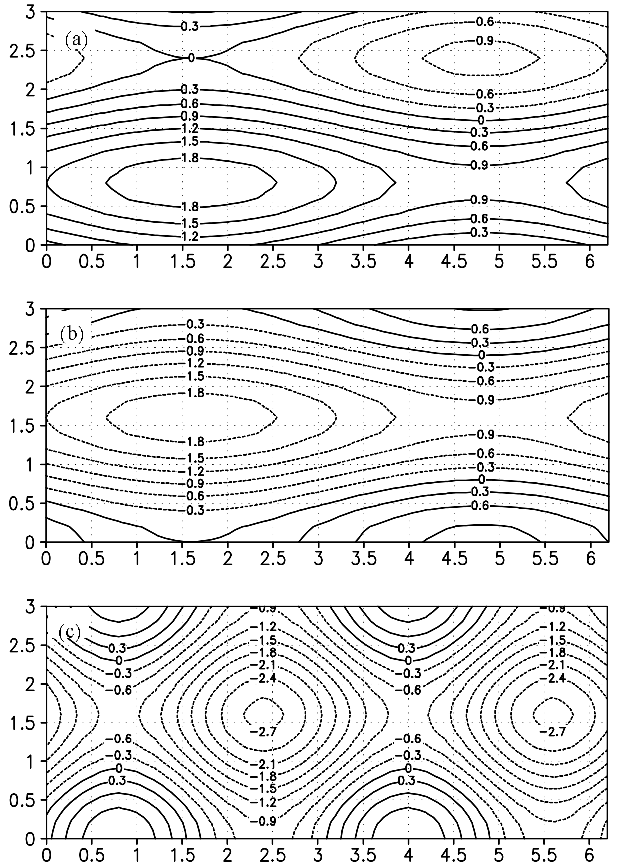

Fig. 1. Three different true initial streamfunction fields, in which the x-axis represents the zonal direction, and the y-axis represents the meridional direction, and the value is dimensionless length (The following Fig. 2 and 5 are the same as Fig.1). (a) Basic 1; (b) Basic 2; (c) Basic 3.

initial state, three cases.

Here, both SVs and CNOPs of the total energy norm are calculated by using the control forecast as the basic state with an optimization time of 48 h, and magnitudes almost equal to that of analysis error. The detailed procedures for computing SVs and CNOPs, and the energy and L_2 norms can be found in MJ (2008). In these 13 different initial control analyses, for some basic states, local CNOPs are found whose objective function is less than that of the global CNOP. However, there is no orthogonality between global and local CNOPs, which is different from the relationships among SVs. Unless specifically stated below, the CNOPs used in the following experiments refer to global CNOPs. A detailed comparison of CNOP and SV for a barotropic model was made by Mu and Zhang (2006), which showed that when nonlinearity is important, use of CNOPs shows great differences from use of SVs. Here, for brevity, only leading SV and

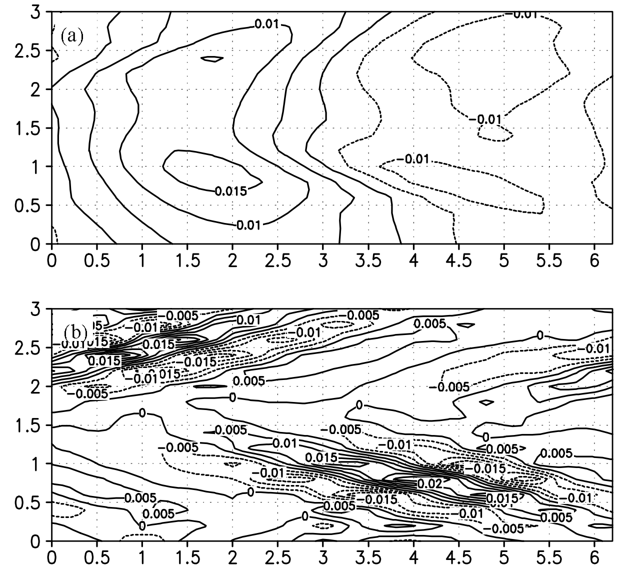

Fig. 2. The streamfunction fields of the first SV and CNOP for the first case. (a) The first SV; (b) CNOP.

CNOP for the first case are presented in Fig. 2, with each having approximately the magnitude of the analysis error, but highlighting the great difference between SV and CNOP. If the analysis errors are slow growing ones, whose nonlinear growth is smaller than that of the SV during the whole forecast time as illustrated in Fig. 3 in MJ (2008), the control run can provide good forecast skill, and the introduction of CNOP may make the forecast skill poorer by comparison. Therefore, for our selected 13 cases, all the analysis errors correspond to fast growing errors, whose nonlinear growth is larger (smaller) than that of SV in the later (early) part of the forecast [similar to Fig. 2 in MJ (2008)]. Thus, it is helpful for us to further discuss the benefit gained by the introduction of CNOP to ensemble prediction for fast growing analysis errors. The type of the analysis error depends on not only the true basic state, but also the distribution of the observation errors. The SV and CNOP schemes construct initial ensemble conditions by adding and subtracting perturbations to and from a control analysis. This procedure of generating perturbation pairs guarantees that the ensemble mean of the initial condition equals the control analysis. Similar procedures are used at the operational centers to minimize ensemble mean forecast error at early forecast leads (Toth and Kalnay, 1993; Molteni et al., 1996). Forecasts from perturbed initial conditions and the control forecast constitute one sample. The ensemble perturbations for sample 1 (S1) are composed of SVs, and for sample 2 (S2), considering that CNOP is the nature extension of the first SV into the nonlinear category, only the first SV

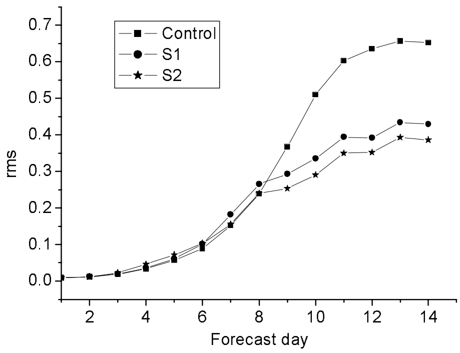

Fig. 3. The root mean squared distance between the ensemble mean forecast and the true state as a function of forecast time.

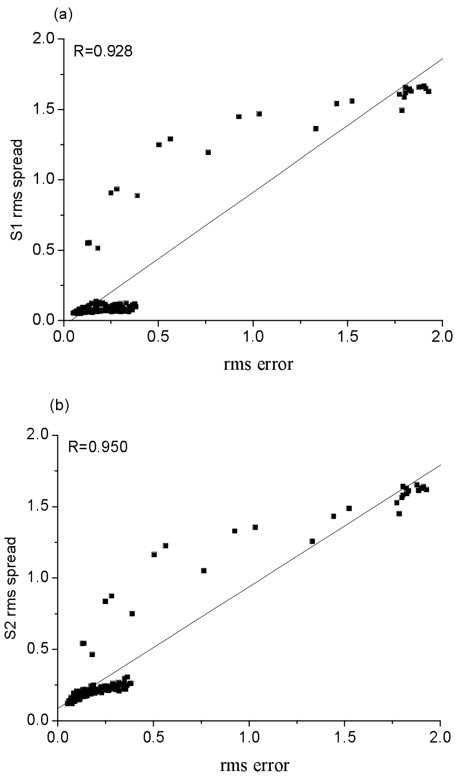

Fig. 4. Spread-skill scatterplots for forecasts of day 6 through day 14, and the corresponding linear correlation coefficients (R), for (a) Ensemble S1 and (b) Ensemble S2.

is replaced by CNOP. Because of the great expense involved in running the model, only a set of relatively small ensembles was be possible in this study. The results in the companion paper have suggested that the use of small ensembles does not present problems in comparing the two types of ensembles. For both ensembles S1 and S2, 7 members were generated, including one control run.

3. Ensemble spread and forecast skill

The relationship between ensemble spread and the skill of the control or ensemble mean forecasts is one of the most basic measures used to verify an ensemble system. One of the principal uses of an ensemble forecast is to provide an estimate of the confidence in a prediction; the larger the ensemble spread is, the less reliable the forecast by any one member is. From this basic notion, it is often assumed that the ensemble spread can be taken as a predictor of the skill of the control forecast. However, even in a perfect environment, spread will not be perfectly correlated with the skill of any individual forecast. It is commonplace to define skill in terms of rms error, which would correspond to choosing an L_2 distance function between a forecast and its verifying analysis. If such a distance function is used to define forecast skill, then for consistency it should also be used in the definition of ensemble spread.

In this section, we first present the rms errors with time between the ensemble mean forecasts and the true state, which is shown in Fig. 3. It is clearly shown that from day 8, both the SV and CNOP techniques improve the forecast skill. Moreover, the CNOP method provides better forecast skill than the SV approach in the medium range (from day 8 to day 14).

Next, we conducted a simple exploration of the relationship between control forecast skill and ensemble spreads by use of scatterplots, which is shown in Fig. 4. Here, the ensemble spread refers to spread with respect to the ensemble mean. Linear correlation coefficients (R) between the forecast skills and ensemble spreads have been computed from day 6 to day 14. For S1, $R = 0.928$, and for S2, $R = 0.950$. This demonstrates that the spread-skill relationhip of S2 appears superior to that of S1 for these 13 cases in the medium range. That is to say, forecasts made by ensemble S2 have higher reliability than those by S1.

To further understand the spread-skill relationship, it is worth investigating the actual spatial distributions of the spread fields and absolute forecast error fields. Figure 5 presents the mean spread fields of S1 and S2 and absolute forecast error fields of the 13 cases. Additionally, the similarity index from MJ (2008) with an L_2 norm is adopted to quantify the degree of similarity between the mean spread and forecast error fields, which is shown in Fig. 6. It can be seen that the absolute forecast error fields are more similar to the spread fields of S2 than to those of S1.

4. Talagrand diagrams

Rank histograms, also known as Talagrand diagr-

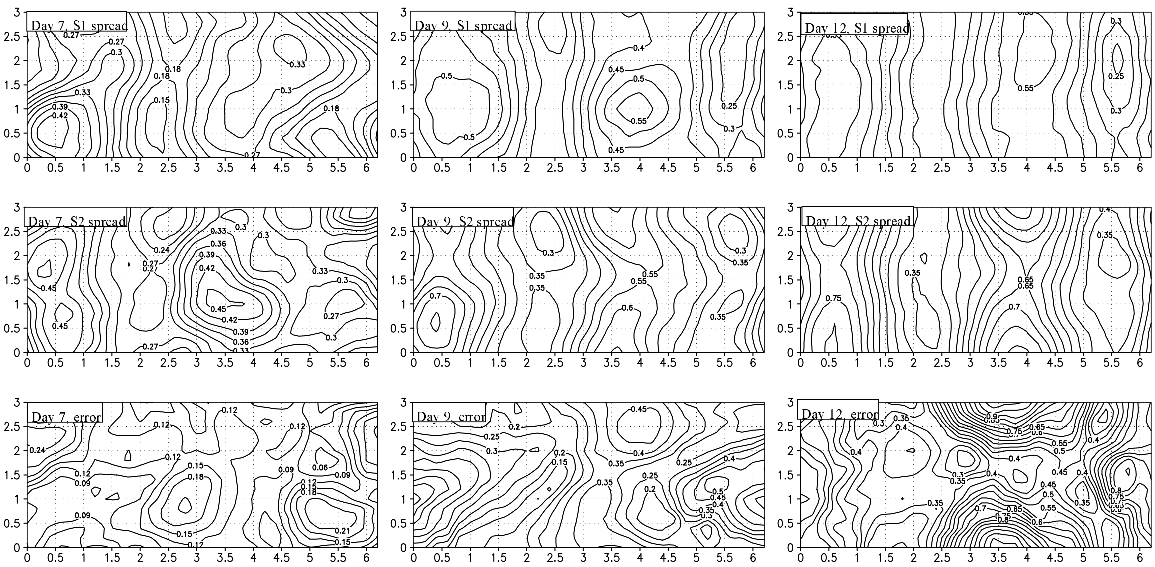

Fig. 5. The ensemble mean spread fields of S1 and S2 and absolute forecast error fields at day 7, 9, and 12.

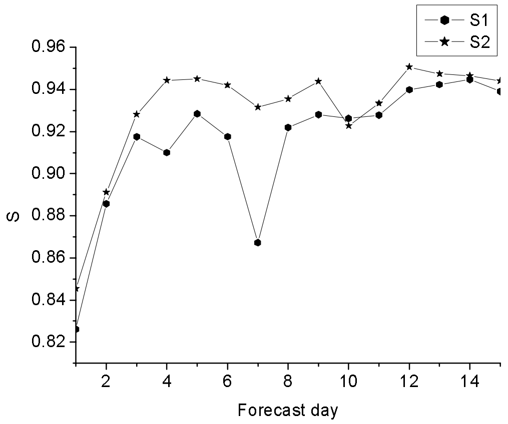

Fig. 6. The similarity index between the absolute forecast error and mean spread fields of S1 and S2.

ams, are a tool for evaluating the reliability of ensemble forecasts (Talagrand et al., 1997). In this section, we investigate the degree of reliability of an ensemble for each perturbation technique using this evaluation method. Uniformity of the rank histogram is a necessary but not sufficient criterion for determining that the ensemble is reliable. Statistics have been accumulated over the above 13 realizations ($L = 13$), and over 512 points, so that the effective sample size is $M = 6656$. Let S_k be the number of elements in the k-th interval of the histogram ($k \in [1, N+1]$). For a reliable system, S_k has expectation $M/(N+1)$. The deviation from flatness of the histogram can be measured by

$$\Delta = \frac{1}{M}\sum_{k=1}^{N+1}\left(S_k - \frac{M}{N+1}\right)^2 , \qquad (4)$$

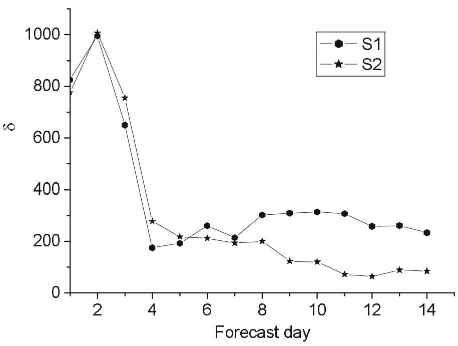

Fig. 7. Evolution of the normalized score δ of the rank histogram as a function of lead time.

where N is the ensemble number, and in this paper $N = 7$. From a perfectly reliable system, Δ is on average equal to $\Delta_0 = N/(N+1)$. Following Candille and Talagrand (2005), we use the ratio

$$\delta = \frac{\Delta}{\Delta_0} \qquad (5)$$

as a measure of the effective flatness of the histogram.

Figure 7 presents the temporal variation of the score δ of the rank histogram. Both the SV and CNOP methods have large initial scores, which decrease significantly over the course of the forecast. At the later lead time, the score for CNOP is smaller than that of SV. The smaller the deviation is, the more reliable the ensemble forecast is. According to this evaluation method, CNOP provides more reliable forecasts than the SV approach from day 6 to day 14.

5. Conclusion and discussions

Over recent years the skills of numerical weather prediction systems have improved considerably. New methods to build initial conditions and refined uses of original perturbation methods have made great contributions.

In our idealized experiments, for fast growing analysis errors, ensembles in which the first SV is replaced by CNOP are superior to the conventional SV method in the medium range forecasts, which can be shown from the spread-skill and Talagrand diagrams. This conclusion further validates the results derived by MJ (2008). Certainly, a study based on just these 13 cases cannot be definitive (Mureau et al., 1993). Moreover, only one CNOP is introduced to the ensemble perturbation; therefore, the question of how to combine different CNOPs or different ensemble perturbation techniques to make better ensemble forecasts is open for further investigation.

In addition, the fast growing errors may correspond to extreme weather events (Oortwijn and Barkmeijer, 1995), which accordingly implies that the CNOP method may be useful in forecasting extreme weather events. The assumption of perfect-model strategy is not a realistic analog for actual numerical weather prediction, where model error may be significant or even dominant. Therefore, use of a global model with real atmospheric observations is needed to test this method for further applications.

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