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Progress in the Study of Nonlinear Atmospheric Dynamics and Predictability of Weather and Climate in China (2007–2011)

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ABSTRACT

Recent progress in the study of nonlinear atmospheric dynamics and related predictability of weather and climate in China (2007–2011) are briefly introduced in this article.

Major achievements in the study of nonlinear atmospheric dynamics have been classified into two types: (1) progress based on the analysis of solutions of simplified control equations, such as the dynamics of NAO, the optimal precursors for blocking onset, and the behavior of nonlinear waves, and (2) progress based on data analyses, such as the nonlinear analyses of fluctuations and recording-breaking temperature events, the long-range correlation of extreme events, and new methods of detecting abrupt dynamical change.

Major achievements in the study of predictability include the following: (1) the application of nonlinear local Lyapunov exponents (NLLE) to weather and climate predictability; (2) the application of condition nonlinear optimal perturbation (CNOP) to the studies of El Niño-Southern Oscillation (ENSO) predictions, ensemble forecasting, targeted observation, and sensitivity analysis of the ecosystem; and (3) new strategies proposed for predictability studies.

The results of these studies have provided greater understanding of the dynamics and nonlinear mechanisms of atmospheric motion, and they represent new ideas for developing numerical models and improving the forecast skill of weather and climate events.

Key words: nonlinear atmospheric dynamics, predictability, weather, climate

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1. Introduction

Nonlinear atmospheric dynamics is an important topic in the study of geophysical fluid dynamics, which itself shows complexity and nonlinearity and presents many important physical and mathematical issues for scientists. The complexities and nonlinearities of atmospheric flows are challenging to analyze and understand or predict. Even though, scientists made significant progresses in the last several decades. For example, the properties of baroclinic instability were extensively studied in linear approximation (Pierrehumbert, 1984; Dimas and Triantafyllou, 1995); the crucial characteristics of the mean state for the development

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of the instability were identified (Charney and Stern, 1962; Bretherton, 1966); the North Atlantic Oscillation (NAO) events were recognized as arising from the eddy forcing in the Atlantic storm track (Vallis et al., 2004); the optimal perturbation method was extensively applied to the studies of large-scale meteorology, oceanography, coupled systems, and even the mesoscale meteorology (Lorenz, 1965; Moore and Kleeman, 1996; Samelson and Tziperman, 2001).

Predictability is a key issue in the study of atmospheric sciences. The complexity, nonlinearity, and randomness of atmospheric motion limit its predictability. Studies on predictability received considerable attention in recent decades due to the pioneering works of Lorenz in the early 1960s (i.e., Lorenz, 1963, 1965, 1969a). One of the great efforts is the exploration of the fundamental limits of predictability (Smith et al., 1999). The predictability of a system is strongly dependent on its stability properties (Moore and Kleeman, 1996; Smith et al., 1999). If a system is particularly unstable, any initial uncertainty that projects significantly onto one of these instabilities will severely limit the skill of an initial-value forecast. Lorenz (1975) showed that the extreme sensitivity of weather predictions to initial conditions means that detailed forecasts are, in general, impossible beyond $\sim$2 weeks. Webster and Yang (1992) demonstrated the "spring predictability barrier" (SPB) of ENSO forecasts by analyzing the correlation between El Niño and the Southern Oscillation. Samelson and Tziperman (2001) showed the growth-phase predictability barrier of El Niño.

Concerning both nonlinear atmospheric dynamics and their related predictability, Chinese scientists have also made an indelible contribution in last few decades. For example, in the study of nonlinear atmospheric dynamics, Mu et al. (1996) established a series of stability criteria for several famous atmospheric models. Luo (2005a–d) proposed a new theory to address the relationship between a blocking flow (planetary scale) and short synoptic-scale eddies. Fu et al. (2003a–c) proposed new methods to solve nonlinear evolution equations, including the Jacobi elliptic function expansion method and the new transformation method. All of these methods benefit our understanding of the irregularities in period and multiple structures of atmospheric motion.

In addition, Chinese scientists explored the calculation of the gradient related to variational data assimilation with "on–off" processes. They presented a new method based on the nonlinear perturbation equation (NPE) for an idealized model to calculate the gradient of the cost function in the presence of on–off switches (Mu et al., 2003; Mu and Zheng, 2005; Wang et al., 2005). In the studies of predictability, Mu et al. (2004) referred it to the study of the uncertainty of forecast results, which consisted of two parts: (1) the analysis of the factors and mechanisms that yield these uncertainties, and (2) the search for methods and approaches to reduce these uncertainties. In addition, the concepts of conditional nonlinear optimal perturbation (CNOP) and the nonlinear local Lyapunov exponent (NLLE) was proposed and used to address the effects of nonlinearity on weather and climate predictability (see the review by Duan et al., 2007); CNOP was also used to study the phenomenon of spring predictability barrier (SPB) for ENSO predictions (Mu et al., 2003). Some new approaches were also developed to improve the predictability of some forecast models (see the review by Duan et al., 2007), which included a new initialization scheme for an intermediate coupled model, the use of the real-time initial atmospheric data in the IAP9L-AGCM, and the use of the reconstruction phase space theory and a spatio-temporal predictive method in a nonlinear dynamical regional prediction model. Furthermore, the ensemble forecast approach was adopted to investigate the predictability of tropical typhoons.

In recent years (2007–2011), Chinese scientists have made further progress in the study of nonlinear atmospheric dynamics and predictability. In this paper, we briefly summarize this progress. In section 2, we review advances in the studies of nonlinear atmospheric dynamics. The progress in the study of predictability of weather and climate are summarized in section 3. Finally, a summary and discussion are presented in section 4.

2. Nonlinear atmospheric dynamics

The basic structures of atmospheric motion can be derived using two methods: (1) controlled equations used to describe the motions of the atmosphere, and (2) patterns derived using datasets of recorded motions of the atmosphere. Thereby, two methods to explore the nonlinearity of the atmospheric motions follow: (1) analyzing the solutions derived from the controlled equations of the atmospheric motions, and (2) analyzing the statistical laws found in the recorded the changes of the atmospheric motions. Progress in the study of the North Atlantic Oscillation (NAO), blocking onset, and nonlinear waves have been achieved using the first method, while progress elsewhere has been obtained using the second method.

2.1 *The dynamics of North Atlantic Oscillations*

The NAO is an important low-frequency dipole mode confined to the Atlantic sector of the Northern Hemisphere (NH). The essential time scale of an NAO event is known to be ~2 weeks (10–20 days; Feldstein, 2000; Benedict er al., 2004; Franzke et al., 2004). NAO events arise from the eddy forcing in the Atlantic storm track (Vallis et al., 2004)); however, many problems such as why the eddy forcing from the Atlantic storm track can drive the NAO event with a life period of ~2 weeks and what determines the phase of the NAO event remain unsolved theoretically.

Luo et al. (2007a,b) developed a weakly nonlinear NAO model to explain why synoptic-scale eddies can reinforce an NAO event with a timescale of 10–20 days and what dominates the phase of a NAO event. Then the weakly nonlinear NAO model was extended to include the effect of topographic waves on NAO events. This generalized model was used to interpret how the zonal mean westerly anomaly changes during the different stage of the NAO event (Luo et al., 2007c). Results showed that the variation of the zonal mean flow is consistent with the change of the NAO amplitude, and the interaction between the NAO anomaly and topographic wave can induce the meridional shift of the zonal mean westerly wind, but the direction of the jet shift is dominated by the phase of the NAO event. In a subsequent paper, Luo et al. (2008a) confirmed that the synoptic-scale wave breaking and the meridional shift of the westerly jet are different descriptions of the NAO phenomenon. At the same time, it was also verified that the phase of the eddy-driven NAO event is determined by the north–south shift of the Atlantic jet prior to NAO onset (Luo et al., 2008b). Moreover, Luo and Gong (2006) also used this weakly nonlinear NAO model to explain why NAO patterns underwent an eastward shift from 1950–1977 to 1978–1997.

More recently, Luo et al. (2010a, b) investigated why the NAO pattern exhibited different spatial structure and zonal shifts during 1958–1977 and 1978–1991. The different tilting of the NAO pattern was due to the different meridional shifts of the center of the Atlantic jet during the different phases of the NAO. Moreover, Luo et al. (2011) found that a very strong Atlantic storm track can result in the transition of the NAO event from the positive to negative phase, thus explaining why the winter mean NAO index underwent a decline during 1991–2009.

2.2 *The precursors for blocking onset*

The dynamics of blocking onset is another important issue in nonlinear atmospheric dynamics. How to determine the perturbations that trigger the blocking onset is very important for forecasting blocking onset. To address this question, Mu and Jiang (2008a) used the linear singular vector (LSV) method and the conditional nonlinear optimal perturbation (CNOP) method to investigate the precursor of blocking onset and to reveal the effect of nonlinearity by a T21L3 quasigeostrophic (QG) model.

CNOP is defined as an initial perturbation, whose nonlinear evolution in a given norm attains the maximum at a prescribed forecast time with physical constraint condition (Mu et al., 2003; Duan et al., 2004). It is a natural generalization of LSV into the nonlinear regime. Jiang et al. (2008) first investigated the CNOPs of a T21L3 QG spectral model. They obtained the CNOPs of the model in terms of three kinds of norms (stream-function-squared norm, total-energy norm, and enstrophy norm) and compared them with their linear counterparts, namely LSVs, revealing the effect of nonlinearity. Results showed that the CNOP method may be a more appropriate tool for the study of stability and sensitivity problems when nonlinearity is of importance. Furthermore, a proper norm should be chosen aiming at different physical problems; based on this, Mu and Jiang (2008a) studied the precursor of blocking onset using CNOP and the T21L3 QG model. At the optimization time of 3 days or 6 days, the CNOP was always the best technique to discover perturbations triggering blocking onset, and the selection of objective function played an important role. For the same initial constraint condition, if a forecast period was extended into the medium range, the advantage of CNOP became more evident.

Furthermore, the inverse problem of determining the precursors to a given blocking anomaly in climatological flow over the Atlantic and Pacific Oceans was explored using the CNOP method with the T21L3 QG model (Jiang and Wang, 2010). A blocking anomaly in geopotential height field is specified as a dipole structure that is dominated by a strong positive anomaly centered at ~60°N and a weak negative anomaly to the south.

Results showed that both for the Atlantic and Pacific blockings, the precursors are baroclinic synoptic-scale wave-train disturbances, whose maximum amplitudes are located in the upstream of the corresponding blocking regions. The disturbances, which mostly focus on the northward flanks of the corresponding Atlantic and Pacific upper-level jets, take on a northeast–southwest trend. However, the leftover parts located in the southward flanks of the corresponding upper-level jet take on northwest–southeast trend. This structure is favorable to the precursors to gain more kinetic energy from the horizontal shear of the basic flow. Further energy analysis reveals that the available potential energy contributes more to the initial precursors, and with time, the kinetic energy dominates the structures. The Pacific block onset is more easily understood from the viewpoint of an eddy-forcing mechanism.

2.3 *Nonlinear waves and their applications*

The NAO and its blockings are different patterns that exist in atmospheric motion that can be described by different simplified controlled equations. Usually, these simplified equations are nonlinear partial differential equations, and they are difficult to solve analytically. However, a number of solvable nonlinear equations exist, such as the Korteweg-de Vries (KdV) equation. It is therefore very important to analytically solve these equations to obtain more and more different analytical solutions, and finally to explain these phenomena. Chinese scientists have proposed a systematic way to find breather lattice solutions to some solvable nonlinear equations (Fu and Liu, 2007; Fu et al., 2007a–c; Zhao et al., 2009). These breather solutions have been used to explain some features of rogue wave (Ruban, 2007). Another way to solve nonlinear equations is to derive their approximate solutions; recently, new Lame functions have been used to derive different multi-order exact solutions of some nonlinear systems (Fu et al., 2009a, b; 2010a, b).

2.4 *Nonlinear analysis of fluctuations in fields related to weather and climate variables*

In this section, we review progress in the nonlinearity of the atmospheric motions which has been explored by analyzing observation data of the atmospheric motion.

Studies by Chinese scientists have shown that statistical similarity in atmospheric motion exists over broad ranges found in a variety of different variables (Chen et al., 2007a; Lin et al., 2007; Lin and Fu, 2008; Feng et al., 2009a; Yuan et al., 2010). Time series of different variables, such as temperature, relative moisture, wind speed, etc. can be used to qualify this kind of statistical similarity, i.e., long-range correlations between values. Different fractal behaviors have been found for different temperature variables (Yuan et al., 2010). Even for a single variable, such as wind speed, the spatial distribution of singularities differs (Feng et al., 2009b), which can be explained by different moments over different scales, where mono-fractal behavior and multi-fractal behavior take obviously different moments using detrended windows. This kind of statistical similarity can be used to define a new index χ to illustrate different characteristics over different climatic regions (Chen et al., 2007b). For example, the northern midland of China and the southern midland of China can be separated by this single index. The inherent physical processes reflected in fluctuations in relative humidity may lead to self-organized behavior, and this may be used to explain the strong power law relationship between observed lightning fire ignition probability and relative humidity. Long-term persistence of different variables where variability is correlated on all time scales can also be used to construct models with long-term persistence to interpret the rapid increase of Earth's temperature.

2.5 *Record-breaking temperatures events in climate changes*

The nonlinearity of the atmospheric motion can also be found from the analysis of the record-breaking temperatures events and from the long-range correlation of extreme events (see section 2.6). Both the theoretic analyseis and Monte Carlo simulation results show that the maximum probability of the occurrence of the kth record-breaking high temperature events tends to increase linearly with the rate $\sqrt{k}\,(k = 1, 2, 3 \cdots\cdots)$, and the frequency of the occurrence of record-breaking high temperature events in a year is inclined to decrease at a rate of $1/(t + 1)$ where t is time. Based on the theory of probability distribution of record-breaking events and daily high/low temperature observation data in China from 1960 to 2005, the spatio-temporal characteristics of record-breaking temperatures were investigated (Xiong et al., 2009). Results showed that the frequency of record-breaking high temperature was obviously greater than normal in Northwest, North, Northeast China, and Tibet, while the frequency of record-breaking low temperature was obviously less than normal there. In recent years, the frequency of record-breaking high temperature events tended to increase in most parts of China, but record-breaking low temperatures became fewer and fewer across China. The strength of record-breaking high temperature events was enhanced in high latitude areas of China, but the strength of record-breaking low temperature events has not changed or has weakened in these areas as well as Xinjiang. Notably, the strength of record-breaking low temperature events has become obviously enhanced in South China.

Based on the monthly Palmer Drought Severity Index (PDSI) of 614 stations in China from 1960 to 2007, the statistics of record-breaking monthly PDSI (RBMP) in these recent 48 years was studied theoretically (Yang et al., 2010). According to the theory of record-breaking events, universal arithmetic regarding the evaluation of record-breaking events was designed. The expression of the expectation value of RBMP was obtained based on the Gaussian distribution model and the initial condition of observed historical RBMP. These numerical results were then compared with those obtained by the iteration computation of the purely theoretical model. The comparison suggested that the results obtained from the former are more consistent with observation data than those from the latter.

2.6 *Long-range correlation of extreme events*

The method of fixed threshold was used to investigate the long-range correlation of extreme events in the Lorenz system (Feng et al., 2009b; Wang et al., 2009). It demonstrated that all of the extreme events with different thresholds exhibit long-range correlation, and the scaling exponents are similar—just smaller than the original series. The results also showed that the long-range correlation of extreme events is less affected by the changes of the initial values, but it decreases distinctly with the increasing parameters (Feng et al., 2009b; Wang et al., 2009). The long-range correlation of Lorenz system's extreme events series has the traits of memory when compared with Gaussian white noise, and the memory is closely related to the threshold. Finally, the maximal daytime air temperature data of 194 stations between 1957 and 2004 from the National Climate Center of China Meteorological Administration revealed that the similar law exists in the practical meteorological factors (Feng et al., 2009b; Wang et al., 2009).

2.7 *Dynamical abrupt change detecting method*

Some new methods have been proposed to detect abrupt dynamic change in time series for the studies of nonlinear atmospheric dynamics. Based on the methods of scaling analysis—detrended fluctuation analysis (DFA), and rescaled range analysis (R/S), He et al. (2008) proposed some new methods to detect abrupt dynamic change in time series, such as the moving detrended fluctuation analysis (MDFA), the moving cut data-detrended fluctuation analysis (MC-DFA), and the moving cut data-rescaled range analysis (MC-R/S). The results demonstrated the validity of these new methods in detecting abrupt change in model time series and observation data. Meanwhile, the results also showed that the window sizes and strong noise have only a tiny effect on the results of MDFA, MC-DFA, and MC-R/S.

Approximate entropy (ApEn) is a valid index that can be used to quantitatively describe the dynamic characteristics and complexity of a time series. ApEn has been developed to detect an abrupt change in one-dimensional time series by sliding a fixed window, which can identify an abrupt dynamic change to some extent. But the sliding ApEn results depend on the window scales, and they cannot accurately position the time-instants of an abrupt change. Based on this limitation, a new method, the moving cut data-Approximate Entropy (MC-ApEn), was proposed by He et al. (2010). This method can be used to detect an abrupt dynamic change in time series. By using this method in model time series, the detected results using the new method have relatively good stability and highly veracity, much better than those using the sliding ApEn method. Also, the application of the new method to daily precipitation records further verified the validity of the new method.

3. Predictability studies for weather and climate

Progress in predictability studies is classified according to three aspects. First, CNOP is applied to ENSO predictions, ensemble forecasting, targeted observation, and sensitivity analysis of ecosystem. CNOP is used in these studies to tackle the issues with the largest forecast errors. Second, nonlinear local Lyapunov exponent (NLLE) is applied to weather and climate predictability. NLLE is used to deal with the limit of the predictable time. Third, new approaches have been proposed for the second kind of predictability studies and for dealing with the lower bounds of the maximum predictable time and the maximum allowable initial errors in the first kind of predictability studies.

3.1 *Applications of CNOP*

3.1.1 *ENSO predictions*

ENSO is a prominent climate phenomenon of the coupled ocean–atmosphere system in the tropical Pacific, and it has a great impact on the global climate. The applications of CNOP to ENSO predictions include the studies of the spring predictability barrier (SPB) and ENSO amplitude asymmetry.

3.1.1.1. The spring predictability barrier (SPB)

The SPB is a universal phenomenon in ENSO prediction. It has been demonstrated that CNOP errors cause a significant SPB for El Niño events, while LSV errors yield a less significant SPB. The non–CNOP-like errors that were studied by Mu et al. (2007b) and Duan et al. (2009) and the random initial errors investigated by Yu et al. (2009) do not induce a SPB. Furthermore, Duan and Zhang (2010) used the model of Wang and Fang (1996) to show that the parameter errors in models may not cause a significant SPB for El Niño events, but initial errors cause a significant SPB. All of these studies indicated that a particular pattern of initial errors is necessary for the SPB of El Niño events to occur. This is consistent with the study of Li and Ling (2009). Actually, there are two types of CNOP errors. One type of CNOP error has an SSTA pattern with negative anomalies in the equatorial central-western Pacific, positive anomalies in the equatorial east Pacific, and a thermocline depth anomaly pattern with positive anomalies along the equator. The other type of CNOP error possesses almost the opposite patterns (Yu et al., 2009). If two such opposite error patterns can be found in realistic ENSO predictions, it may establish the robustness of the characteristic of the initial errors that causes a significant SPB and demonstrates the dominant role of the initial errors in the SPB.

Two types of initial errors suggest two dynamical behaviors of error growth in the SPB phenomenon for El Niño events. The initial error patterns, which have a dynamical behavior similar to El Niño and La Niña, may exhibit an apparent season-dependent evolution. This may cause a large prediction error, which yields a significant SPB. Two classes of CNOP-type errors possess almost the same dynamical behavior as El Niño and La Niña and induce the largest prediction error; thus, they can be regarded as most likely to cause a significant SPB phenomenon. The two types of LSV errors also have the same dynamical behavior as El Niño and La Niña; however, they cause a less significant SPB, due to a much more localized spatial region as compared to the CNOP's. These results encourage us to consider whether the forecast skill for ENSO can be greatly improved when these types of initial errors are filtered out through data assimilation or targeted observation approaches.

In addition, the CNOP-type errors cover a broader region than the LSV-type errors. In addition, this localized region of the CNOP-type errors, which have large values always arising in the equatorial central-eastern Pacific, is more likely to capture the "sensitive area" of ENSO prediction. This result may guide efforts to intensify observations in this area and improve ENSO prediction.

3.1.1.2. ENSO amplitude asymmetry

ENSO amplitude asymmetry is an important problem in ENSO studies (Jin et al., 2003). A better understanding of ENSO amplitude asymmetry contribute to better prediction of ENSO. ENSO asymmetry is regarded as the phenomenon in which the amplitude of the observed El Niño is larger than that of La Niña. This is a distinct feature of ENSO. Furthermore, there is evidence that ENSO amplitude asymmetry has become pronounced since the climate shift around the year 1976, from a relatively stable to an unstable oscillating system (Duan and Mu, 2006). Duan et al. (2004), An and Jin (2004), and Rodgers et al. (2004) used a nonlinear ENSO system respectively to study ENSO amplitude asymmetry; they demonstrated consistently that ENSO amplitude asymmetry is a typical nonlinear property of the coupled ocean–atmosphere system. Furthermore, Duan and Mu (2006) demonstrated that nonlinearity induces ENSO asymmetry. The stronger the ENSO events are, the stronger the nonlinearities are, and the more significant the ENSO asymmetry is.

Consequently, Duan et al. (2008) used the CNOP approach to investigate the roles of different types of nonlinearities in ENSO asymmetry, and they revealed the decisive role of nonlinear temperature advection. They adopted both the theoretical model developed by Wang and Fang (1996) and the intermediate Zebiak-Cane model. The results of the two models are consistent, that is, the nonlinear temperature advection considerably enhances El Niño amplitude and trivially affects La Niña amplitude, causing ENSO amplitude asymmetry. The strong ENSO asymmetry with strong nonlinearity of strong ENSO events indicates why ENSO asymmetry becomes strong after the 1970s. That is to say, the decadal change of ENSO asymmetry may be due to the change of nonlinearity. As such, a CNOP approach could be extended to study the decadal variability of ENSO.

3.1.2 *Ensemble forecasting*

Ensemble forecasting, which serves as an effective way to improve the weather forecast, has been classified into the regime of predictability studies (Mu et al., 2004). One of the key problems in ensemble forecasting is the generation of initial ensemble perturbations, which are expected to reflect the real initial uncertainty. At the European Centre for Medium-Range Weather Forecasts (ECMWF), the LSV approach has been successfully applied to generating initial perturbations for ensemble forecasting. However, the linear theory of SV could not guarantee the optimum result with a nonlinear system. Considering this point, Mu and Jiang (2008b) used CNOP to construct the initial perturbation fields for ensemble forecasting, in an attempt to remedy the limitation of LSV and then improve forecast skill.

Under a perfect model assumption, Mu and Jiang (2008b) demonstrated that the ensemble forecast skill when using CNOP may depend on the type of the analysis error. When the analytical error is a fast-growing type, the CNOP initial perturbation field, which is obtained by replacing the first SV of the perturbation field yielded by SVs with CNOP, causes a better ensemble mean forecast than the SV-type initial perturbation field composed of SVs. Furthermore, with the reduction of the magnitudes of analysis error, the ensemble mean skill caused by the CNOP-type initial perturbation field approaches gradually that caused by the SV-type initial perturbation field. This indicates that a CNOP initial perturbation field could capture more effectively the characteristic of the fast-growing type of analytical error, consequently making the ensemble forecast better. When the analytical error is a slow-growing type, the Monte Carlo method can provide a good forecast, whereas the ensembles mean of the CNOP- and SV-type initial perturbation field make the forecast worse.

3.1.3 *Targeted observation*

Targeted observation, which places observations in specific regions (sensitive areas) according to weather or climate events such as tropical cyclones, precipitation, etc., is another important technique used to improve predictability. Palmer et al. (1998) first applied the LSV approach in targeted observation. Due to the limitations of LSV approximation, the sensitive area determined by LSV may be questionable. It is necessary to use a nonlinear technique to determine the sensitive area in targeted observation. Therefore, Mu et al. (2007c) and Mu et al. (2009) respectively investigated primarily the application of CNOP in targeted observation for precipitations and typhoons.

Results showed that the structures of CNOPs may differ greatly from those of FSVs depending on the constraint, the metric, and the basic state. CNOP errors have larger impacts on the forecasts in the verification area as well as the tropical cyclones or the precipitations than the FSV errors. The results of sensitivity experiments indicated that reductions of CNOP errors in the initial states provide more benefits than reductions of FSV errors. These results suggest that it is worthwhile to use CNOP as a method to identify the sensitive areas in adaptive observation for precipitation or tropical cyclone prediction.

Subsequently, Zhou and Mu (2011, 2012a,b), Qin and Mu (2011, 2012), and Chen and Mu (2011) further confirmed the utility of CNOP in targeted observations by investigating (1) the impact of verification area design and (2) the impact of horizontal resolution on the CNOP-identified sensitive areas; (3) the time-dependence issues of the CNOP sensitive areas; (4) the influence of CNOP sensitivity on typhoon track forecasts; (5) comparisons among the CNOP, SV, and ensemble transform Kalman filter (ETKF)-identified sensitive areas; and (6) the analysis of the structure and distributions of the growth errors. Wang and Tan (2009) developed a fast algorithm to solve CNOP and then use the CNOP to identify the sensitive areas of typhoon precipitations. The results were inspiring because the forecast of typhoon precipitation had been largely improved by assimilating the observations in the sensitive areas.

For typhoon-targeted observations, another dynamic method was proposed by Gao et al. (2009), in which the sensitive areas were determined by invoking the negative anomalies of moist potential vorticity.

3.1.4 *Sensitivity analysis of ecosystem*

CNOP has been applied in sensitivity and stability analyses of baroclinic unstable flow and ocean circulation. Mu and Wang (2007) extended the application of CNOP to the sensitivity analysis of grassland ecosystem, in an attempt to reveal the effect of nonlinearity on the transition between grassland and desert states. Previously, a three-variable theoretical model developed by Zeng et al. (1994) and Zeng and Zeng (1996) had been adopted.

The results showed that the moisture index μ plays an essential role in the grassland ecosystem. When μ is less than the first bifurcation point μ_1, desert equilibrium state (DES) is nonlinearly stable, even for the large initial finite-amplitude perturbations, which implies that the ecosystem is droughty and nonlinearly stable. It is impossible to change the desert state into a grassland state just by planting grass or irrigating. When the moisture index μ is larger than the second bifurcation point μ_2, the grassland equilibrium state (GES) is conditionally nonlinearly stable. That is, there exists a threshold value of initial perturbation (denoted by δ), $\delta = 1.04795$, which represents the mass density of living grass ($\bar{x}$) in the grassland equilibrium state. When the magnitude of an initial perturbation δ is smaller than $\bar{x}$, there is no initial perturbation to cause a transition to DES; but for the case of $\delta \geqslant \bar{x}$, if a destructive action is made such that the value of the living grass component of the initial perturbation is null, the ecosystem will evolve to DES. This case suggests that it is still important to keep the balance for the ecosystem even if the soil is washy and the natural condition is feasible. When μ is between μ_1 and μ_2, the grassland ecosystem is fragile, GES or DES is linearly stable but nonlinearly unstable, meaning that a large enough initial finite-amplitude perturbation can induce a transition from GES to DES or DES to GES, respectively. The management of human activities is important when moisture index μ is in (μ_1, μ_2).

To explore the nonlinear features of the ecosystem, Mu and Wang (2007) also calculated LSVs of GESs and DESs. Comparisons between their nonlinear evolutions demonstrated that for the same magnitude using CNOP and LSV, CNOP is more likely to yield a transition than LSV.

These studies applied CNOP to estimate the sensitivity of ecosystem. Especially, CNOP was used to tackle the problem of the transition between equilibrium states. The transition between equilibrium states is a typical nonlinear property. A finite amplitude initial perturbation superimposed on an equilibrium state will cause the state transition to another equilibrium state. The application of CNOP to this field could help elucidate the mechanism of the transition between equilibrium states.

3.2 *New approaches in predictability*

3.2.1 *An extension of CNOP and its applications*

The CNOP approach was proposed by Mu et al. (2003) to study predictability from the viewpoint of error growth. Previously, CNOP was used to determine the optimal initial perturbation (CNOP-I) in a given constraint, and it was used to investigate the first predictability problem for weather and climate (Mu et al., 2007a, b; Duan et al., 2009). However, the existing numerical models have not yet precisely described atmospheric and oceanic motion without model errors (Williams et al., 2005; Orrell, 2003). Effect of model errors on predictability is related to the second kind of predictability (Lorenz, 1975). One important aspect in this field is the study of the effect of the uncertainties of model parameters on predictability (Lu and Hsieh, 1998; Mu, 2000; Mu et al., 2002). Some scientists investigated the effect of model parameter errors on the predictability of a numerical model by choosing a control parameter and considering the different perturbations on this control parameter (e.g., Chu, 1999) or by taking different values of each model parameter and exploring the effect of the uncertainties of the parameters on climate simulation (e.g., Zebiak and Cane, 1987; Orrell, 2003). However, in realistic predictions, multiple parameters of models may simultaneously have uncertainties; moreover, model parameter errors may be accompanied by initial errors. Estimating the predictability limit caused by these combined error modes has far-reaching consequences for the atmospheric study and forecast and should be pursued urgently.

Mu et al. (2010) established an objective function consisting of initial perturbation and model parameter perturbation, and they extended the CNOP approach to search for the optimal combined mode of initial perturbations and model parameter perturbations. This optimal combined mode, also named as CNOP, has two special cases: CNOP-I only links with initial perturbations and has the largest nonlinear evolution at prediction time; whereas CNOP-P is related to parameter perturbations that cause the largest departure from a given reference state at prediction time. The CNOP approach facilitates the exploration of not only the first kind of predictability related to initial errors but also the second kind of predictability associated with model parameter errors. Moreover, CNOP can be used to address the predictability problems of the coexistence of initial errors and parameter errors. CNOP was used to study ENSO predictability in a theoretical ENSO model. The results demonstrated that the prediction errors caused by the CNOP errors are only slightly larger than those yielded by the CNOP-I errors and that model parameter errors may play a minor role in producing significant uncertainties in ENSO prediction. Therefore, CNOP errors and their resultant prediction errors illustrate the combined effect on predictability of initial errors and model parameter errors. They represent the relative importance of initial errors and parameter errors in yielding considerable prediction errors and they can help to identify the dominant source of the errors that cause prediction uncertainties.

3.2.2 *New strategies of solving a class of optimization problems related to predictability*

In 1975, Lorenz classified two kinds of predictability problems (Lorenz, 1975): Initial error with an assumption of perfect model is referred to as the first kind of predictability. Model errors with a perfect initial field comprise the second kind of predictability. The former has been extensively investigated, and many theories and methods have been proposed or introduced (e.g., Lorenz, 1965; Toth and Kalnay, 1997; Mu et al., 2003; Mu and Zheng, 2006; Riviere et al., 2008) in which optimal methods are important for estimating the limit of the predictability of weather and climate events (e.g., Lorenz, 1965; Fan and Chou, 1999; Smith et al., 1999; Mu, 2000; Mu et al., 2003). According to the practical demands of weather and climate predictions, Mu et al. (2002) classified three predictability problems: the maximum predictability time, the maximum prediction error, and the maximum allowable initial error and parameter error.

The second predictability problem, i.e., maximum prediction error, can be solved using an existing highly efficient solver for computing CNOPs. However, for the first and third predictability problems, although Mu et al. (2002) and Duan and Mu (2005) solved these using a filtering method with two very simple ordinary differential equation models of two or three dimensions, it was impossible to solve them when a large-scale system of high dimensionality was considered. To effectively solve these two predictability problems, Duan and Luo (2010) introduced a strategy for solving for the lower bounds of the maximum predictable time and the maximum allowable initial errors of the first kind of predictability. This strategy was compared to an existing filtering method. A series of comparisons showed that the results of the new strategy were almost the same as those of the old method; furthermore, the new methods saved a large amount of computation time. The most advantageous aspect of the new strategy was the computation of the largest prediction errors caused by initial errors in a given constraint. The largest prediction error in the new strategy was calculated using an existing optimization solver, which had been verified to be highly efficient, whereas in the old method, the largest prediction error was filtered out by comparing the prediction errors caused by the initial errors in the given constraint. Therefore, using the old filtering method was impossible for solving the above two predictability problems in a complex and realistic model, but the new strategy provided the opportunity to study and possibly solve these two predictability problems for a realistic weather and climate models.

3.2.3 *Ensemble forecast experiments of ENSO events*

Although significant progress has been made in ENSO theories and predictions in recent years, there is considerable uncertainty in ENSO prediction, even at relatively short lead times, due to the chaotic or irregular aspects of climate variability. As a result, forecasts must necessarily include some quantitative assessment of this uncertainty, to make up the deficiency of the deterministic forecast.

A new large size (i.e., 100 members) ensemble prediction system (EPS) has been developed recently to make ensemble ENSO forecast routinely in real time with useful skills reaching up to 2 years (Zheng et al., 2006, 2007, 2009a; Zheng and Zhu, 2008, 2010a). To minimize forecast uncertainties for ENSO predictions, the EPS is primarily based on an intermediated coupled model (ICM) and is secondarily based on the ensemble Kalman filter (EnKF) data assimilation method. This coupled method was adopted to generate the initial ensemble conditions for the EPS through assimilating all available atmospheric and oceanic observations using a developed coupled data assimilation scheme (Zheng and Zhu, 2010a). In addition, a linear, first-order Markov stochastic model-error model was embedded within the EPS to represent the model uncertainties during the 12-month ensemble forecast process (Zheng et al., 2009a).

A 16-year retrospective forecast experiment (November 1992 to October 2008) showed the deterministic prediction skill of the EPS had been significantly improved compared to the original deterministic forecast scheme (Zhang et al., 2005). This improvement occurred because the advanced assimilation method can provide more dynamically consistent and accurate initial conditions than the original initialization method (Zheng and Zhu, 2010a), and the ensemble mean can remove some unpredictable stochastic information (Zheng et al., 2009b). At the same time, the spring predictability barrier (SPB) can be alleviated in a probabilistic sense through reasonably considering the impacts of model uncertainties on ENSO's seasonal predictability (Zheng and Zhu, 2010b).

3.3 *Nonlinear local Lyapunov exponent (**NLLE**) and its application in predictability*

The approach of the nonlinear local Lyapunov exponent (NLLE) was introduced to study the predictable time in atmospheric predictability from the view of nonlinear error growth dynamics (Li et al., 2006; Chen et al., 2006; Ding and Li, 2007). Based on the NLLE, Ding and Li (2007) obtained the saturation theorem of the mean relative growth of initial error (RGIE). Using the saturation theorem, the NLLE and its derivatives can be used to quantify the predictability limit of chaotic dynamical system. For systems whose equations of motion are explicitly known, such as the Lorenz system, the mean NLLE via numerical integration of the system and its error evolution equations can be directly calculated (Ding and Li, 2007; 2008a). The NLLE approach has been used to investigate some predictability problems of the Lorenz system, such as relative effects of the initial error and the parameter error on predictability (Ding and Li, 2008b), the relationship between the limit of predictability and initial error (Li and Ding, 2011a), and the predictability limits of different variables in multidimensional chaotic systems (Li and Ding, 2009).

Atmospheric observation data contain almost all of the real information regarding the day-to-day movement and evolution of weather systems. Given that the precise dynamical equations of atmospheric motion are explicitly unknown, it is more appropriate to investigate atmospheric predictability based on observation data. To apply the NLLE in studies of actual atmospheric predictability, an algorithm based on local analogues was devised to enable the estimation of the NLLE and its derivatives using experimental or observation data (Ding et al, 2008; Ding and Li, 2009a; Li and Ding, 2011b). The general idea of the algorithm is to find local analogs of the evolution pattern from observational time series. The local analogs are searched for based on the initial and evolutionary information at two different time points in the time series. If the initial distance at two different time points is small and if their evolutions are analogous over a very short interval, it is highly likely that the two points were analogous at the initial time. This analog is referred to as a "local dynamical analog".

As noted by Lorenz (1969b), a sufficiently long time series is required when using historical analogs to study atmospheric predictability. It is almost impossible to find good natural analogs within current libraries of historical atmospheric data over large regions such as the Northern Hemisphere. However, it should be noted that the "local dynamical analog" searches the observational time series for a small local region, for which the small number of spatial degrees of freedom makes it possible to find good local analogs within current libraries of historical atmospheric data, which allows an ensemble average (van den Dool, 1994). In contrast to a "global analog" or "spatial pattern analog" over a large region, the "local dynamical analog" yields information on local predictability, thereby providing the spatial distribution of the limit of local predictability.

One example of the NLLE algorithm from the Lorenz96 40-variable model (a low-order proxy for an atmospheric model; Lorenz, E. N.) reveals that the algorithm is entirely applicable in estimating the mean error growth from an experimental time series (Li and Ding, 2011b). The NLLE algorithm can be further applied to studies of atmospheric predictability because the global attractor exists in the atmosphere (Li and Wang, 2008). Based on atmospheric observation data, the NLLE approach has been used to investigate the temporal–spatial distributions of weather predictability (Ding and Li, 2009b), decadal changes in weather predictability (Ding et al, 2008), the spatio-temporal distribution of the predictability of monthly and seasonal means of climate variables (Li and Ding, 2008), the predictability limit of the MJO (Ding et al, 2010, 2011), and the spatio-temporal distribution of the predictability of sea surface temperature (SST; Li and Ding, 2011c).

For weather predictability, the results show that for the daily 500-hPa geopotential height field, the limit of weather predictability appears to have a zonal distribution, with a maximum limit of 10–14 days over the tropics and Antarctic, followed by 8–11 days over the Arctic, 6–11 days over the middle to high latitudes of the Northern Hemisphere; the lowest limit of 4–6 days is in the mid-latitude of the Southern Hemisphere. The vertical distributions of the predictability limit of the daily geopotential height show an increase in predictability limit with height. The fact that the predictability limit is <3 weeks in the troposphere and is ~ 1 month in the lower stratosphere indicates that the stratosphere may be used as a potential predictability source (Ding and Li, 2009b).

For decadal changes in weather predictability, the results show that significant decreasing trends in the weather predictability limit (WPL) could be found in most regions of the northern mid-latitudes and Africa, while significant increasing trends in WPL lie in most regions of the tropical Pacific and southern middle-to-high latitudes. Trends and interdecadal changes of the WPL are found to be well related to those of atmospheric persistence, which in turn are linked to the changes of atmospheric internal dynamics. Further analyses indicate that the changes of atmospheric static stability due to global warming might be one of main causes responsible for the trends and interdecadal changes of atmospheric persistence and predictability in the southern and northern middle-to-high latitudes (Ding et al, 2008).

For climate predictability, the results show that the predictability limit of monthly and seasonal means have obvious differences between the tropics and middle to high latitudes. The predictability limit of monthly and seasonal means is the largest in the tropics, and it decreases quickly from the tropics to the middle-to-high latitudes of the Southern and Northern hemispheres. In the tropics, the predictability limit of monthly means is >6 months, with the maximum value >9 months, and the predictability limit of seasonal means is >8 months, with the maximum value >11 months. However, in middle-to-high latitudes, the predictability limit of monthly means is only 2–3 months, and the predictability limit of seasonal means is only 4–5 months (Li and Ding, 2008).

For the predictability of the MJO, the results show that the predictability limit of the MJO, as determined by the NLLE approach, is ~ 5 weeks, which exceeds the performance of most numerical and statistical prediction models (Ding et al, 2010). The potential predictability limit of the boreal summer intraseasonal oscillation (BSISO) is close to 5 weeks, comparable to that of the boreal winter MJO. Despite the similarity between the potential predictability limits of the BSISO and MJO, the spatial distribution of the potential predictability limit of the TISV during winter is approximately opposite that during summer. The error growth is rapid when the BSISO and MJO enter the decaying phase (i.e., when ISO signals are weak), whereas it is slow when convection anomalies of the BSISO and MJO are located in upstream regions (i.e., when ISO signals are strong) (Ding et al., 2011).

For the predictability of the monthly SST, results show that the annual mean limit of SST predictability is the greatest in the tropical central–eastern Pacific (>8 months). Relatively high values were also obtained for the tropical Indian and Atlantic Oceans (5–8 months). In the northern and southern middle-to-high latitude oceans, the limit of SST predictability is <6 months, with a minimum value of only 2–3 months. The limit of SST predictability in different ocean areas shows significant seasonal variations, related to the persistence barriers that occur during particular seasons. These seasonal persistence barriers cause a relatively low limit of SST predictability when predictions are made across the season in which the barriers occur. In contrast, when predictions are initiated from the season with a persistence barrier, the SST errors show rapid initial growth but slow growth in the following seasons, resulting in a relatively high limit in predictability (Li and Ding, 2011c).

4. Discussion and conclusions

Progress in the study of nonlinear atmospheric dynamics and related predictability in China during 2007–2011 were summarized in this article. For nonlinear atmospheric dynamics, Chinese scientists developed a weakly nonlinear NAO model, explained why synoptic-scale eddies can reinforce a NAO event with a time scale of 10–20 days, and investigated what dominates the phase of a NAO event. In addition, the optimal precursors for blocking onset were indentified by a quasi-geostrophic model using the CNOP method. In addition, Chinese scientists made contributions in nonlinear analyses of fluctuation and wave dynamics related to atmospheric motions and in understanding the record-breaking temperatures events in climate changes, long-range correlation of extreme events, and dynamical abrupt change within time series. For predictability studies, CNOP was extended to consist of not only initial perturbation but also parameter perturbation, and CNOP can be used to explore both the first kind of predictability and the second kind of predictability. Several applications of the CNOP were reviewed, such as the SPB for El Niño events, ENSO irregularity, the ensemble forecast, the targeted observations, and the sensitivity of the ecosystem. A new nonlinear technique NLLE was developed and used in predictability studies, revealing the predictability limit for several weather and climate phenomena. In addition, a new strategy was developed to solve the nonlinear optimization problems of maximum predictable time and maximum allowable initial errors and parameter errors, suggesting the possibility of studying these two predictability problems in a realistic model.

As shown, Chinese scientists have conducted many studies of nonlinear atmospheric dynamics and related predictability studies during the period of 2007–2011. The results of these studies can be applied to guide scientists to improve numerical modeling and to increasingly understanding the dynamics of atmospheric and oceanic motions. The combination of theoretical studies and practical applications can lead to the improvement of forecast skill for weather and climate, in particular, extreme weather and climate events.

However, we are still far away from exposing the nature or cause of complex nonlinear phenomena. Some important problems remain for future study and improvement. For example, in the study of NAO, blocking, and ENSO prediction, current results are based on simple models; whether the results hold in more complex systems is also an interesting issue. In those studies related to the applications of CNOP, the optimization algorithm needs to be improved to more efficiently calculate CNOP. In targeted observation for tropical cyclone prediction, the design of a proper cost function is urgent. In the study of the second kind of predictability, the approaches associated with model errors need improvements. In the NLLE studies, further work is required to examine broader applications of the NLLE method in predictability studies of extreme weather and climate events, the prediction of the predictability, and ensemble forecast, and so on. Much more progress is expected in the future.

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