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Handling Error Propagation in Sequential Data Assimilation Using an Evolutionary Strategy

BAI Yulong*[1,2] (摆玉龙), LI Xin[1] (李 新), and HUANG Chunlin[1] (黄春林)

[1] *Cold and Arid Regions Environmental and Engineering Research Institute, Chinese Academy of Sciences, Lanzhou* 730000

[2] *College of Physics and Electrical Engineering, Northwest Normal University, Lanzhou* 730070

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ABSTRACT

An evolutionary strategy-based error parameterization method that searches for the most ideal error adjustment factors was developed to obtain better assimilation results. Numerical experiments were designed using some classical nonlinear models (i.e., the Lorenz-63 model and the Lorenz-96 model). Crossover and mutation error adjustment factors of evolutionary strategy were investigated in four aspects: the initial conditions of the Lorenz model, ensemble sizes, observation covariance, and the observation intervals. The search for error adjustment factors is usually performed using trial-and-error methods. To solve this difficult problem, a new data assimilation system coupled with genetic algorithms was developed. The method was tested in some simplified model frameworks, and the results are encouraging. The evolutionary strategy-based error handling methods performed robustly under both perfect and imperfect model scenarios in the Lorenz-96 model. However, the application of the methodology to more complex atmospheric or land surface models remains to be tested.

Key words: data assimilation, error propagation, evolutionary strategies

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1. Introduction

Atmospheric and other similar models (e.g., land, hydrological, and oceanic) still have large but unknown deficiencies. This uncertainty comprises one of the major obstacles to atmospheric research because large unknown errors can seriously undermine modeling results (Li et al., 2007; Reichle, 2008). Poor error estimation or parameterization can lead to filter divergence (Jazwinski, 1970), a situation in which the filter becomes so overconfident around an incorrect state that the subsequent observation data are ignored and the estimate cannot be moved back toward the true state (Anderson, 2007; Li et al., 2009; NG et al., 2011). A practical way to address this issue is through a localization or inflation technique. In Ensemble Kalman Filter data assimilation, localization is used to modify the error covariance matrices to suppress the influence of distant observations, removing spurious long-distance correlations (e.g., Houtekamer and Mitchell, 2001; Greybush et al., 2011; Tian and Xie, 2012). Much effort has been devoted to advancing the state-of-the-science in data assimilation for better inflation techniques. Some experimental research has been performed regarding the atmospheric and oceanic data assimilation (DA) field, such as covariance or multiplicative inflation (Anderson and Anderson, 1999), additive inflation (Hamill et al., 2005), and the "relaxation-to-prior" method proposed by Zhang et al. (2004). All of these methods for dealing with model errors are meant to alleviate the bias error in ensemble second moment. As an extended application of the maximum likelihood theory developed in the works of Dee (1995) and Dee and da Silva (1999), Zheng (2009) proposed a "multivariate covariance inflation" to extrapolate the inflation factor to a time-dependent diagonal matrix. However, only a simple model and independent observation errors were tested in that study. Liang et

*Corresponding author: BAI Yulong, yulongbai@gmail.com

al. (2011) further developed the work of Zheng (2009) and they used spatially correlated observation errors to test the inflation method on more realistic models. Motivated by the "relaxation-to-prior" method (Zhang et al., 2004), Bai and Li (2011) proposed a new method to generate the ensemble perturbation based on the crossover principles originally developed in intelligent computing research. The essence of the method is to generate proper ensemble perturbation to inflate the covariance matrix and to obtain appropriate weights for innovation in the ensemble Kalman filter by which filter divergence can be effectively mitigated. Combined with the factor search frameworks, the most ideal error adjustment factors guaranteed the best DA performance in the corresponding circumstances. The preliminary results from idealized cases suggested its potential for data assimilation. Nevertheless, the experiments also indicated that the crossover principle suffers from high computational costs due to its complex conceptual formulation and relative difficulty of implementation.

Subsequent to the work of Bai and Li (2011), the main purpose of this study was to develop a new parameterization method using evolutionary strategies (hereafter, ES, cf, Recehenberg, 1965), which is both relatively easy to achieve by programming and is computationally efficient. Basically, ESs are population-based metaheuristic optimization algorithms that use biology-inspired mechanisms like mutation, crossover, natural selection, and survival of the fittest to refine a set of solution candidates iteratively (Back et al., 1997; Fogel, 2006). Technically, crossover is an operator applied to two or more selected candidates (the so-called parents) that results one or more new candidates (the children). Mutation is applied to one candidate and results in one new candidate. Crossover and mutation lead to a set of new candidates (the offspring). With the aim of facilitating implementation, the emphasis in this study was to develop evolutionary strategies inspired by natural mutation and natural selection concepts in biological evolution that are more computationally efficient. Mutation operation in evolutionary strategies is more important than in the genetic algorithms (GAs) proposed by Bai and Li (2011). In addition, the algorithms generally operate directly on the actual values that are to be optimized, in contrast with the GAs, which usually operate on a separately coded transformation of objective variables. Therefore, the evolutionary strategy proposed in this work is simple in concept, has brief parameterizations, and is easy to implement. This strategy has been proven to be an efficient method to solve optimization problems, and it has been successfully applied in the areas of function optimization, neural network training, and fuzzy control systems (Back et al., 1997; Whitley, 2001; Lee et al., 2006). Finally, we chose an evolutionary strategy to address error estimation issues in response to the suggestions of Whitaker et al. (2008: pp477): "One can either add more tunable parameters to the parameterization to force the structures to match, or try to develop new parameterization that more accurately reflect the structure of the underlying system error covariance."

This paper is organized as follows: in section 2 the methods are described, the study results are discussed in section 3, and in section 4 conclusions are presented.

2. Methods

In this study, ES principles were applied to sequential deterministic filters. After a one-step assimilation, each column of the analysis ensemble perturbation was taken as the initial population of the ES algorithms. The size of the initial population was the same as the number of ensembles. Each individual of the population was real-coded to calculate its fitness value. Based on the principles of Darwinian evolution, the population was optimized by choosing genetic operators (e.g., inheritance, mutation, selection, and recombination). The offspring after the evolution were taken as the final analysis ensemble perturbation into the next step of assimilation. In this section, the DA methods are briefly described, including evolutionary strategy-based error parameterization methods and error factor searching methods.

2.1 *Data assimilation methods*

Based on the ensemble transform Kalman filtering method (ETKF, Bishop et al., 2001), a new error parameterization method coupled with ES was developed. The method takes each ensemble of the ETKF as an individual in the evolutionary algorithms, while the integrations of ensembles are taken as the evolution of the individual. This study may be considered a continuation of the previous work by Bai and Li (2011) because the same ensemble data assimilation approach (ETKF) and the same model were used. [For more details about the DA methods, refer to Bai and Li (2011).] After a one-step assimilation, all ensemble perturbation matrices are taken into account. The analysis ensemble perturbation matrix is denoted as an $m\times N$ -dimensional matrix $\boldsymbol{X}_{\rm a}$ [cf. Eq. (13) of Bai and Li, 2011], where m is the model dimension and N is the number of ensembles. Elements in each column of $\boldsymbol{X}_{\rm a}$ represent the ensemble disturbance between ensemble analysis and the ensemble mean state. The ith column element is $\boldsymbol{X}_{\rm a}(i)=\{\boldsymbol{x}_{\rm a}(i)-\bar{\boldsymbol{x}}_{\rm a}\}$, where $\bar{\boldsymbol{x}}_{\rm a}$ is the analysis mean state and $\boldsymbol{x}_{\rm a}(i)$ is the analysis state after the assimilation.

In this study, the analysis ensemble perturbation $\boldsymbol{X}_{\rm a}(i)$ was further investigated to address filter divergence problems. Although all KF-based algorithms assume that the perturbations should satisfy Gaussian distribution, DA systems may violate this assumption after several steps of assimilation. Therefore, the ES principle has been introduced to obtain a more appropriate perturbation with the constraint of the fitness function by adjusting crossover and mutation factors. This method represents an alternative to traditional methods, such as multiplicative methods and additive methods, which are meant to change the ensemble perturbation to respond to a lack of Gaussian conformity caused by the nonlinear model during the assimilation process. To take the above perturbation as the parents' individual perturbation, evolutionary techniques are applied. The offspring $\boldsymbol{X}_{\rm a,o}(i)$ (the 'o' in the subscript means the offspring) after the evolutionary action are taken as the final analysis ensemble perturbation into the next assimilation step. Finally, the model state vector at time t_n is the summation of the assimilation mean and the evolutionary offspring [cf. Eq. (15) of Bai and Li (2011)].

$$\boldsymbol{x}_{\mathrm{a},\,\boldsymbol{t_n}} = \bar{\boldsymbol{x}}_{\rm a} + \boldsymbol{X}_{\rm a,o}(i)\ . \tag{1}$$

2.2 *Evolutionary strategy-based error parameterization methods*

Error parameterization methods proposed in this work use the analysis ensemble perturbation $\boldsymbol{X}_{\rm a}(i)$ as the initial population, which is denoted as $\boldsymbol{X}_{\rm a}$ for simplicity. Based on Schwefel (1981), the evolutionary algorithm is allowed to self-adapt the vector of standard deviation σ appropriate for each parent. Each evolving trial solution is encoded not only with the vector of object variables $\boldsymbol{X}_{\rm a}$ to be optimized but also with a vector of standard deviation σ that in part determines how $\boldsymbol{X}_{\rm a}$ and σ are mutated into $\boldsymbol{X}_{\rm a,o}$ and $\sigma_{\rm o}$. Specifically, for all components $i = 1 \ldots N$, N is the ensemble size.

The following steps describe the evolutionary algorithm proposed in this work:

(1) The generation of the initial population. The individual is composed of the target valuable $\boldsymbol{X}_{\rm a}$ and the standard deviation $\boldsymbol{\sigma}$. Each part is divided into N components as shown:

$$\begin{aligned}(\boldsymbol{X}_{\rm a}, \boldsymbol{\sigma}) = [&(\boldsymbol{X}_{\rm a,1}, \boldsymbol{X}_{\rm a,2}, \ldots, \boldsymbol{X}_{{\rm a},i}, \ldots, \boldsymbol{X}_{{\rm a},N}),\\ &(\sigma_1, \sigma_2, \ldots, \sigma_i, \ldots, \sigma_N)]\ ,\end{aligned} \tag{2}$$

The relationship between $\boldsymbol{X}_{\rm a}$ and σ is the following:

$$\begin{cases} \sigma_{{\rm o},i} = \sigma_i \cdot \exp[\lambda_1 \cdot N(0,1) + \lambda_2 \cdot N_i(0,1)] \\ \boldsymbol{X}_{{\rm o},i} = \boldsymbol{X}_{{\rm a},i} + \sigma_{{\rm o},i} \cdot N_i(0,1) \end{cases}\ , \tag{3}$$

where $(\boldsymbol{X}_{{\rm a},i}, \sigma_i)$ is the ith components of the parent individual; $(X_{{\rm o},i}, \sigma_{{\rm o},i})$ is the ith components of the offspring individual; $N(0,\ 1)$ is an evenly distributed random number with the range [0, 1] that is generated for each mutation per individual; $N_i(0,\ 1)$ is also an evenly distributed random number with the range [0, 1], but this number is generated for each σ_i. λ_1 is the global parameter equal to $\left(\sqrt{2\sqrt{N}}\right)^{-1}$; λ_2 is the local parameter equal to $(\sqrt{2N})^{-1}$; and N is the size of the population. These equations indicate that the offspring are obtained randomly based on the parent individuals.

(2) Generation of the new population. According to evolutionary strategy, the new population is generated by convex crossover between the population of assimilation analysis and the population after the evolution. In our algorithm, several improvements to the convex crossover operator have been made:

$$\boldsymbol{X}_{\rm a,o}(i) = (1 - e_1) \times \boldsymbol{X}_{{\rm a},i} + e_1 \times \boldsymbol{X}_{{\rm o},i}\ , \tag{4}$$

in convex crossover operation, error adjustment factor e_1 is the random number with the uniform distribution within (0, 1). The value of the error adjustment factor determines the accuracy of the DA systems. The offline searching process is equivalent to bringing the feedback mechanism to the DA system to obtain more ideal results.

(3) Repeat steps 1 and 2. To take the standard deviation σ and the convex crossover error adjustments factor e_1 as the feedback factors, the best error adjustment factor (σ, e_1) is calculated to yield the most ideal final result during data assimilation.

2.3 *Error factor searching methods coupled with GA*

To handle error adjustment factor searching problems, a coupled forward-inverse approach was developed, implemented, and tested by Bai and Li (2011). The coupled approach was formulated using ETKF and the genetic algorithm. The offline method adopts an outer optimization algorithm to find appropriate error adjustment factors (σ, e_1).

3. Numerical experiments

We investigated the performance of the methods we developed in a number of experiments with small toy models. We conducted two experiments, starting with the 3-element Lorenz (Lorenz-63) model from the work of Lorenz (1963), followed by the 40-element Lorenz (Lorenz-96) model from the work of Lorenz (1996).

3.1 *Experiments with the Lorenz-63 model*

The Lorenz-63 model consists of a system of three coupled and nonlinear ordinary differential equations (Lorenz, 1963):

$$\begin{cases} \dfrac{dx}{dt} = \alpha(y-x) + q_x \\ \dfrac{dy}{dt} = rx - y - xz + q_y \\ \dfrac{dz}{dt} = xy - bz + q_z \end{cases} , \quad (5)$$

where $\alpha = 10, r = 28, b = 3/8$. $x(t), y(t)$ and $z(t)$ are the dependent variables. The terms q_x, q_y, and q_z are assumed to represent the unknown model errors. As described in Evensen (2007), the terms were set as 2, 12.13, and 12.31 amplitudes of Gaussian white noise to simulate model errors.

3.1.1 *The fitness function*

Fitness function is the method used to determine the performance of each individual of the population in the genetic algorithm. This is the only way to achieve the selection step in the genetic algorithm. We used the root-mean-square error of the analysis state ($\mathrm{RMSE_a}$) to evaluate the accuracy of the assimilation results. The target of the search was the minimum of the fitness function. Although the fitness function imported here uses the truth in toy models and this new DA method is limited for now to synthetic studies, the choice of the fitness function in actual applications has been thoroughly discussed in Bai and Li (2011). The analysis RMSE is defined as the following:

$$\mathrm{RMSE_a} = \sqrt{\frac{1}{N}\left[\sum_{i=1}^{N}(x_{\mathrm{a},i} - x_{\mathrm{s},i})^2\right]} , \quad (6)$$

where N is the number of the ensemble, $x_{\mathrm{s},i}$ is the truth, and $x_{\mathrm{a},i}$ is the analysis value.

3.1.2 *Experimental results*

In this study, the genetic algorithm was used to consider with the second moment error in ETKF. Therefore, the application process of ES was investigated according to the ensemble DA. The ES algorithms (see section 2.2) were implemented. Coupled with the fast-searching genetic algorithm discussed in Bai and Li (2011), the most ideal error adjustment factors were obtained with the constraints of fitness function. After the evolution, the algorithm used the best offspring to propagate forward until a new observation value was met. By updating the error adjustment factors adaptively, the entire procedure continued until the GA's end conditions were met.

3.1.3 *Evolutionary strategy principles experiments*

The application process of ES is presented in this section. In Bai and Li (2011), a feasible solution space for DA systems was defined. Figure 1 shows the schematic of the application with 10 ensembles. The mutation and crossover operations search all of the points in the lineage of the two parents can easily be seen. Therefore, the capacity of ES for wide searching is more ideal than that of the convex crossover operator (Bai and Li, 2011).

3.1.4 *Sensitivity experiments of evolutionary strategy principles*

To evaluate the ES DA systems, the ensemble sizes, observation windows, and observation error magnitudes were all varied. The standard experiment parameters were set as the following: initial value x_0=[8.0 0.0 30.0], time increment Δt=0.01, observation windows w= 8, observation error R_o=[2 2 2], and ensemble number N=3. All parameters except one

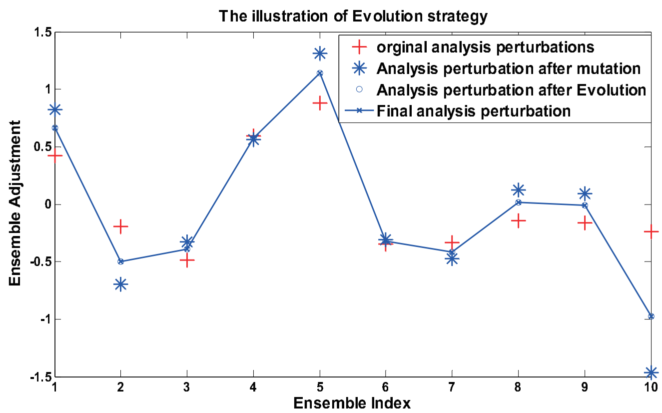


Fig. 1. The schematics of ensemble analysis perturbations after evolutionary strategy operation to the ensemble number N=10.

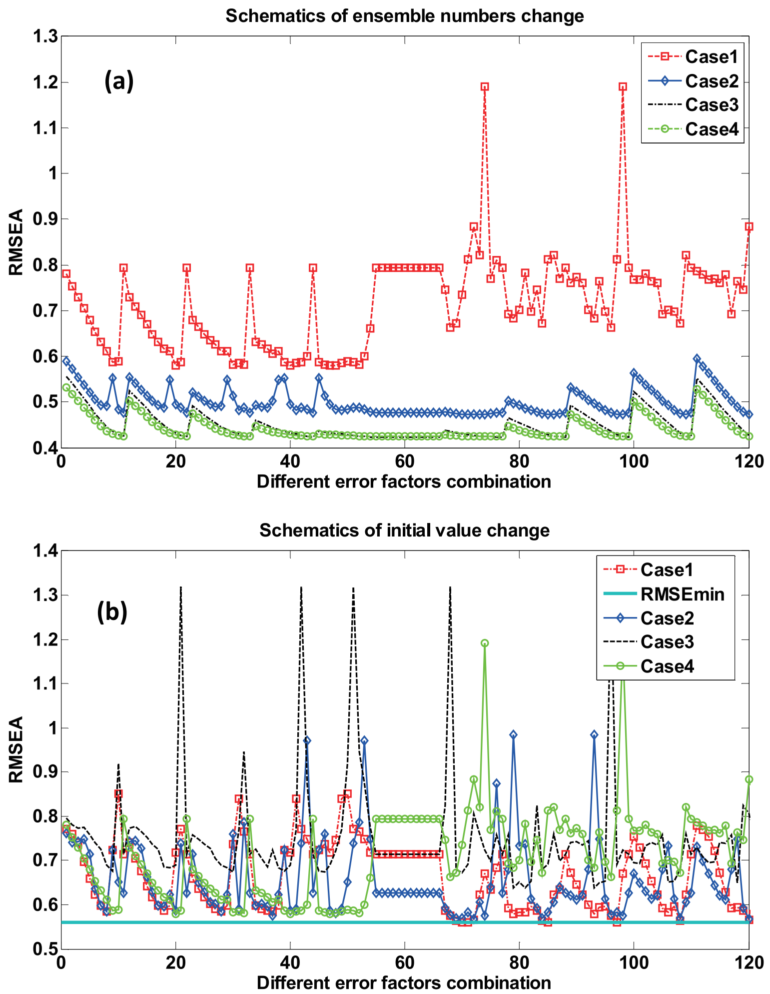


Fig. 2. The assimilation results with different ensemble numbers and different initial values. "Different error factor combination" means different values of σ and e_1. (a) Results with different ensemble number, and (b) results with different initial value. With the different error factor combinations, the RMSE changes rapidly with an evident multiple hill and multiple valley phenomenon, which can be taken as evidence that the target functions are convex functions.

were fixed each time. Using this method, the influence of evolution was studied to analyze the function of each parameter. Meanwhile, the correctness of the ES methods can be further proven according to the normal Lorenz model DA experiment, which has been studied thoroughly by many researchers (Evensen, 2007; Kalnay et al., 2007). Four sets of parameters were considered (Table 1):

(1) The influences of the ensemble numbers. In this experiment, only the ensemble sizes were changed, and other parameters were maintained. Different error factor combinations (σ, e_1) were applied, and the results indicate the following (Fig. 2a): (a) With the increase of the ensemble sizes, the RMSE of the assimilation system decreased, which is in accord with the result of the standard Lorenz model experiment. If the ensemble number increased to 10, the change was no longer apparent (Kalnay et al., 2007). (b) When the ensemble number was smaller, the amplitudes of RMSE change appeared to change dramatically with changes in the error factor. With the increase of the ensemble number, the amplitude tended to change smoothly. (c) With the different error factors combination (σ, e_1), the RMSE changed rapidly with evident multihill and multivalley phenomena, which can be taken as evidence that the target functions are convex functions. Therefore, search methods should be applied to obtain the most ideal combination.

(2) The influence of initial value. As seen in Fig. 2b, the initial values yielded small changes from case to

Table 1. The parameters for the DA sensitivity experiments with the Lorenz-63 model.

	Initial value			Observation covariance				
	X	Y	Z	X	Y	Z	Observation interval	Ensemble numbers
Case-1	1.5	−1.5	25	1	1	1	4	3
Case-2	3.5	−3.5	27	2	2	2	8	6
Case-3	5.5	−5.5	29	4	4	4	12	10
Case-4	9.5	−9.5	33	8	8	8	20	20
Case-5	8	0	30	10	10	10	25	30

Table 2. The best RMSE and optimal error adjustment factors (σ: mutation factor; e_1: crossover factor; RMSE$_m$: the minimum of RMSEa).

	Initial value			Observation interval			Initial value			Initial value		
	σ	e_1	RMSE$_\mathrm{m}$	σ	e_1	RMSE$_\mathrm{m}$	σ	e_1	RMSE$_m$	σ	e_1	RMSE$_\mathrm{m}$
Case-1	0.1	0.3	0.5594	0.1	0.5	0.4903	−0.3	0.9	0.4605	−0.4	0.8	0.5795
Case-2	0.1	0.5	0.5667	−0.4	0.8	0.5795	−0.4	0.8	0.5795	0.5	0.9	0.4723
Case-3	0.2	0.3	0.6374	0.2	0.3	0.6994	−0.2	0.2	0.7706	0	0.7	0.4237
Case-4	−0.4	0.8	0.5795	−0.5	0.1	1.0035	−0.2	0.5	1.0316	0.1	0.8	0.4243
Case-5	0.2	0.2	0.6046	0.5	0.2	1.3287	−0.3	0.5	1.1178	−0.1	0.9	0.4233

case. However, the multiple hill characteristic of the minimum solution of RMSE with the different cases was more apparent. Therefore, combined with the fast-searching genetic algorithm, the most ideal error couples can be determined to acquire the most ideal RMSE results of each case.

(3) The influence of observation covariance. The results of assimilation changed with observation covariance (Fig. 3a). When the observation covariance is increasing, the RMSE of assimilation systems increases significantly as well, which is in accord with the conclusion of the normal assimilation methods (Evensen, 2007; Kalnay et al., 2007).

(4) The influence of the observation interval. The RMSE of assimilation changed with the changes of error adjustment factors (Fig. 3b). The results indicate the following: (a) With the increase of the observation interval, the RMSE of assimilation systems increased significantly, in accord with the normal results (Evensen, 2007; Kalnay et al., 2007). (b) When the observation intervals change, more ideal results can be obtained when the crossover error factor is 0.5 (i.e., the crossover operation emphasizes the balance between the analysis ensemble perturbation and the mutation perturbation). (c) The increase of the observation interval causes the amplitude of RMSE to change dramatically.

Table 2 shows the schematic of the most ideal RMSE with the most ideal mutation and crossover factors. The results indicate the following: (a) As for different cases, the positions of the best mutation and crossover factors differ, which means that ES gives different weights to analysis ensemble perturbation and mutation perturbation. The most ideal analysis smooths the mutation value and analysis value, which is in accord with the "relaxation-to-prior" method proposed by Zhang et al. (2004). (b) With the increase of the ensemble number, the most ideal error factor gives more weight to the analysis value. When the ensemble number increases to a certain value, the most ideal assimilation result tends to be a constant value.

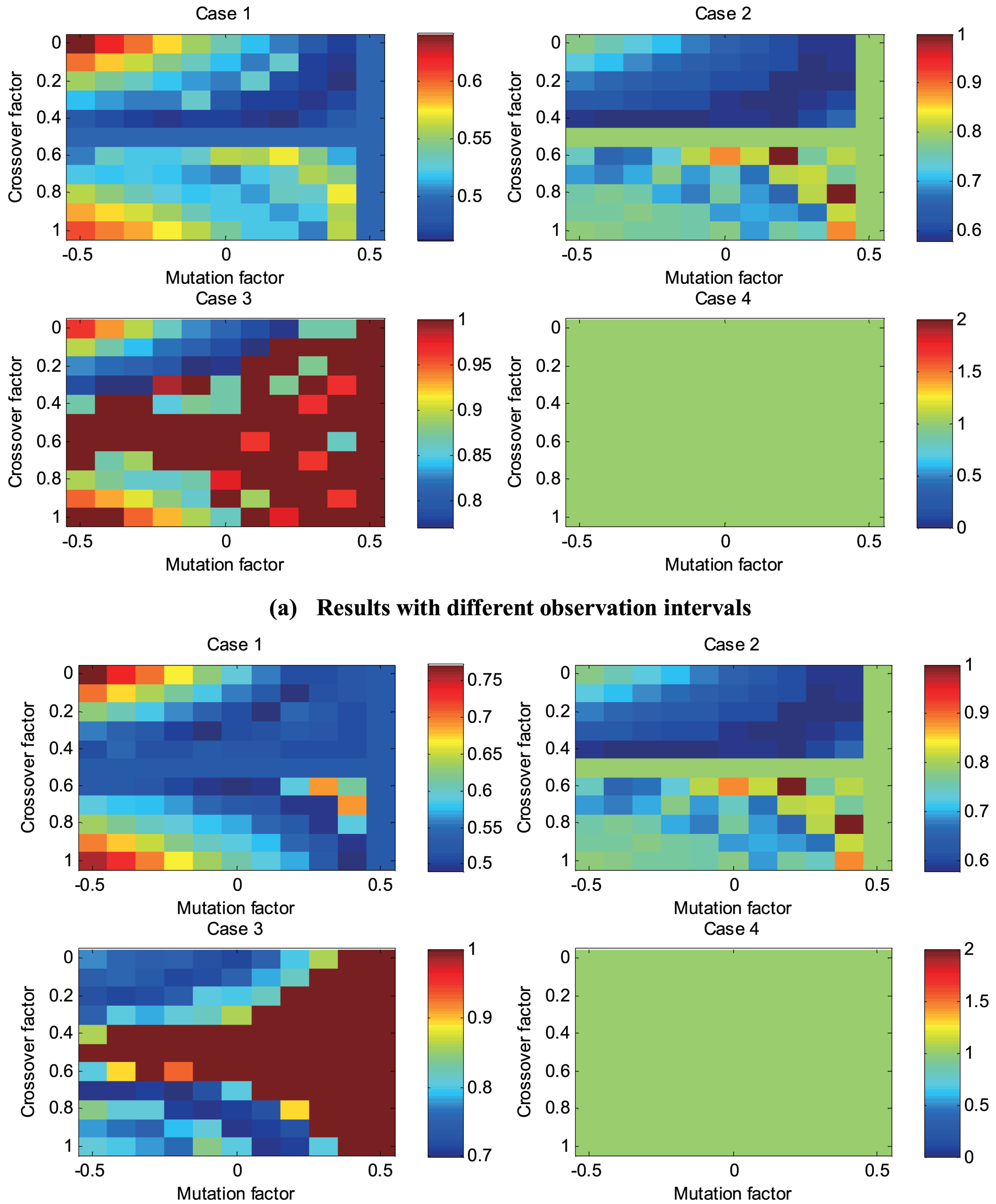
Figure 4 shows the schematic of RMSE change, which is identical to the standard conclusions drawn by other researchers (Evensen, 2007; Kalnay et al., 2007). The results indicate the following: An increase of the ensemble number significantly reduces the assimilation error. An increase of observation covariance and observation interval increases assimilation RMSE. The changes of initial value of the model bring little change to the results.

3.2 *Evaluations with the Lorenz-*96 *model*

To detect the larger effects of the proposed assimilation technique, we conducted experiments with the 40-element Lorenz-96 model. The Lorenz-96 model is a toy model originally proposed in the context of atmospheric dynamics that is used extensively to test novel techniques and applications. It is a time-continuous model consisting of a set of nonlinear ordinary differential equations coupled in a ring geometry, which uses the following formula:

$$\frac{dx_l}{dt} = -x_{l-2}x_{l-1} + x_{l-1}x_{l+1} - x_l + F \, . \tag{7}$$

Here, $l = 1, 2, \ldots, 40$. $x(t)$ can be seen as a scalar meteorological variable. The boundary is cyclic, i.e.,



(b) Results with different observation variances

Fig. 3. The assimilation results with different observation intervals and different observation variances. (a) Results with different observation intervals, and (b) results with observation variances. The color brown means "Filter Divergence" and denotes the regions with RMSEs >1.0.

$x_{-1}=x_{l-1}$, $x_0=x_l$, and $x_{l+1}=x_1$. This model behaves chaotically in the case of external forcing F=8. In this study, Eq. (7) was solved using the Runge-Kutta fourth-order scheme with an integration time step of 0.01. Thus, five steps correspond to six hours.

In the Observation System Simulation Experiments (OSSEs), the Lorenz-96 model was set up first. Then we created some bogus observations to assimilate. To detect the performance of the proposed methods in a more complex model, we varied ensemble size, number of observations, and observation magnitude as we had previously. All of the results obtained from the experiments were the same as those of the ordinary DA system (Khare et al., 2008). When the ensemble number increased, the accuracy of the DA system increased, but the running time became longer. When the observation interval and observation error covariance were large, the accuracy of the DA system de-

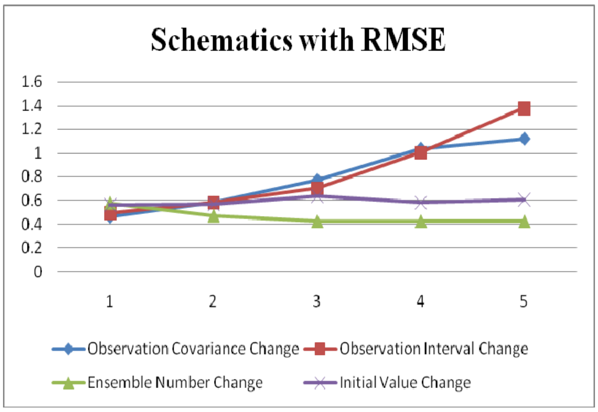


Fig. 4. The best error factor variations and RMSE for the cases given.

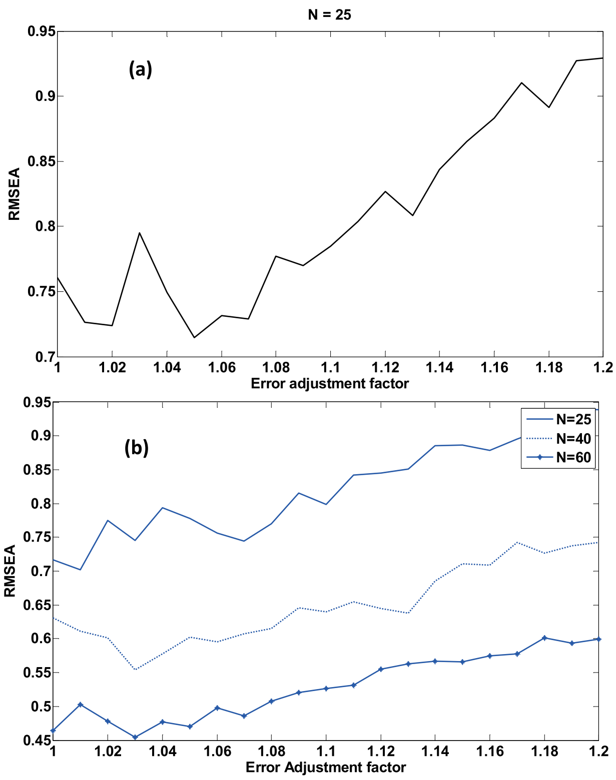


Fig. 5. RMSE changes with different error factors (a) The multiple hill characteristics of the solution (ensemble size N = 25) and (b) the significant increase of the RMSE error when the ensemble size increases (ensemble size N = 25, 40, 60). When the fast-searching method was applied, the smallest RMSE for each case resulted.

creased.

To illustrate the influence of evolutionary strategies proposed in this study, we use a typical example from the ensemble numbers variations. In this configuration, the standard experimental parameters were set thus: the ensemble number N=25, observational number m=16, and the observational error covariance $R_{\rm o}$=2. Figure 5 gives the RMSE of the assimilation result changes when the error adjustment factor changed. The following characteristics were revealed: (1) A better filter performance can be obtained within the best range of 1.0–1.2. Beyond this range, the optimization is no longer obvious. (2) Within the best numerical range, the obvious multiple hill and multiple valley characteristics of the target destination functions can be seen (Fig. 5a). The best error adjustment factor can be detected using the proposed evolutionary strategies. (3) With the increase in the ensemble number, the RMSE of the assimilation systems significantly decreased (Fig. 5b). This was also a finding of Khare et al. (2008). All of the results obtained here were compared with those of Khare et al. (2008) to confirm their accuracy.

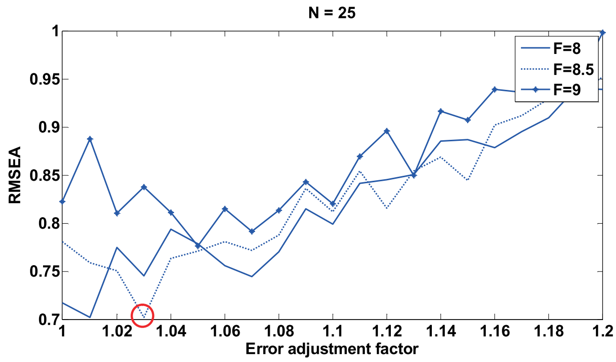


Fig. 6. When the error adjustment factors changed, the RMSE of the assimilation also changed. The results are shown for different parameter setups with the Lorenz-96 model. (1) without model error (F=8), (2) with moderate model error (F=8.5), and (3) with severe model error (F=9).

To examine the sensitivity of the proposed methods to the variations of the different model error assumptions, we designed another group of experiments with different experiments parameters based on the work of Tian et al. (2011). The performance of assimilation systems were compared under the perfect-model assumption (F=8 for all truth, forecast, and assimilation runs), a different (incorrectly specified) forcing coefficient (F=8.5), and the severe model error (F=9) (Fig. 6). The experimental configurations were exactly the same as those for the perfect model case. Notably, in the presence of several kinds of model errors, the results indicate the following: (1) The trend of the error adjustment factors is the same as the trend of the forcing coefficients. When the model error increases, the RMSE of the assimilation systems significantly increases. (2) In each case, the multiple hill and multiple valley characteristics show that better assimilation results can be obtained using the proposed evolutionary strategies. (3) When the forcing coefficient is 8.5, the same assimilation results as those obtained for a perfect-model assumption can be achieved by searching for the best error factors (Fig. 6, red circle). Therefore, the DA methods coupled with the evolutionary strategy can achieve the same results when the model error is large; the evolutionary strategy is capable of outperforming the normal DA methods under both perfect and imperfect model scenarios with lower computional costs. The method is robust even when the forecast model contains a significant bias error, as is confirmed by the proposed method when the perfect model was replaced by the imperfect one.

4. Summary and concluding remarks

In this study, considering the suggestions of Whitaker et al. (2008), a new scheme was developed to improve the evolutionary algorithm-based error parameterization methods proposed by Bai and Li (2011). Applying the evolutionary strategy, offspring were generated through the typical procedure using the mutation and crossover operations. According to the premise of maintaining the original assimilation mean value, the offspring was added as the updated analysis information to propagate forward. Because mutation factors and crossover factors are considered in ESs to control the evolution position and model performance, the use of ESs can not only optimize the ensemble population but can also bring feedback mechanisms to the DA system (Bai and Li, 2011). Coupled with fast-searching genetic algorithms, the most ideal error adjustment factors may be retrieved. Several numerical experiments performed with the Lorenz-63 model and the Lorenz-96 model showed that our method performed better than the original ones. Compared with traditional error processing methods such as multiplicative inflation methods and addictive methods, DA systems, based on evolutionary strategy, can to some extent explain the inner physical laws of the error processing and propagating in ensemble data assimilation. The preliminary results suggest the potential performance of our model for real atmospheric assimilation: it provides a promising new method for data assimilation, especially for its applicability to some new ensemble-based 4Dvar methods (e.g., Wang et al., 2010; Tian et al., 2011; Tian and Xie, 2012)

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