Comments on Zeng's Paper "Variational Principle of Instability of Atmospheric Motions"

Zeng Qingcun (曾庆存)
LASG, Institute of Atmospheric Physics, Academia Sinica, Beijing 100080
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After my paper (Zeng, 1986b) was published and another (Zeng, 1989) was submitted to the journal, I found two papers written by Arnold (1966) and McIntyre et al. (1987) and received some reprints of Ripa's papers (1983; 1984; 1987; 1988) in the same field. I thank Drs. Mu Mu and Pedro Ripa very much for showing and sending me these interesting papers.

According to Arnold (1966) the function $Q(q)$ determined by the basic flow $\bar{\Psi}$ can be defined in $\bar{q}_{min} \leqslant q \leqslant \bar{q}_{max}$ and extrapolated to outside of this interval with continuous derivatives of needed orders. Therefore, Theorem 2.2 in Zeng's paper (1989) can be stated more precisely as follows.

Theorem 2.2 A (basic) flow $\Psi(\theta, \lambda - \dot{\lambda}_0 t)$ determined by $\delta I = 0$ with a function $Q(q)$ and parameters r_0, r_1, and r_2 is always stable with respect to every small perturbation if either $r_1 Q'' \equiv 0$ holds everywhere, or $r_1 Q''$ and r_0 are of the same sign in the fluid except for a domain of zero-measure where $r_1 Q'' = 0$.

Similar improvement of the statement can be introduced to Theorem 5.2 concerning the three-dimensional quasi-geostrophic model.

All conservations of energy, "generalized enstrophy", mass, angular momentum, and "generalized boundary energy" are included in the invariant functional in Zeng's works, but only the first and second conservations are considered by Arnold and McIntyre et al.. The "generalized boundary energy" exists in the three-dimensional quasi-geostrophic model with idealized linear or proper nonlinear boundary condition at the bottom surface of the fluid. The more comprehensive consideration leads to Zeng's results (1986b; 1989) applicable to unsteady basic flows (propagating waves) and overcoming the difficulty arising in McIntyre et al.'s paper with the nonlinear boundary condition.

A prior estimate technique was applied by Arnold and McIntyre et al. to reduce the domain of instability. Zeng's results can be also improved by this technique and extended to the study on structure instability (see Mu and Zeng, 1988).

The problem with an infinitive channel without periodicity or without finite total energy in a β-plane is only dealt with in Zeng's work (1987; 1989) using proper modification of the Liapounoff's functional and the variational technique.

Criteria of nonlinear stability or instability of motions in the models governed by the primitive equations are dealt with in Zeng's (1986b; 1989) and Ripa's (1983; 1988) papers by using the variational principle. In general, the criteria obtained in barotropic one-layer and multiple layer models are the same except for the rigid lid approximation included in Ripa's paper. In barotropic one-layer model the criteria are almost the same as the one obtained by using linearized governing equations (see Zeng, 1979; 1986a; Ripa, 1983) if the depth of the fluid is large and the perturbation small enough, otherwise the domain of instability in the nonlinear case might be very much extended as indicated by Zeng (1989). Ripa made more analyses and applications of his results to the oceanographic dynamics (Ripa, 1983; 1984; 1987; 1988), while Zeng gave physico–mathematical classification of the instabilities and more corollaries (inferences and remarks) (Zeng, 1986a; 1989).

The continuous three–dimensional model governed by baroclinic primitive equations is the most difficult. By considering whether the limit $N \to \infty$ is possible, where N is the number of layers of the layer model, Ripa (1988) gives some hints that it is difficult to find sufficient condition for the stability. While, by directly dealing with the continuous baroclinic model Zeng (1989) indicates that the second–order variation is always not a positive–definite functional and that the Helmholtz instability exists whenever there is a vertical shear of the basic flow.

Finally, unlike other authors, Zeng (1986b; 1987; 1989) points out that there exist whole family of propagating waves such as linear and nonlinear Rossby–Haurwitz waves by requiring that the first–order variation equals zero. In fact, these waves which are the solutions of the non–divergent or geostrophic model have been given in Zeng's paper (1989), and those which are the solutions to the primitive equations constructed and verified by numerical integrations (Zeng, 1979; Zhang and Zeng, 1983; Zeng et al., 1985; Zhang et al., 1986; 1987).

Besides, some printed errors in Zeng's paper (1989) should be corrected as follows:

1. page 139, line 1, sinq should be replaced by $\cos\theta$.
2. page 143, line 20; the correct name is Hoskins.
3. page 145, line 11, λ_2 should be replaced by $+\lambda_2$.
4. page 145, Fig.2. The first two lines from above should correspond to $\delta^2 I / 2r_0 < 0$ and $\delta^2 I / 2r_0 = 0$ respectively.
5. page 155, line 4, delete Θ.
6. page 156, line 4, Frm = 1 should be replaced by Frm < 1.
7. page 158, line 18, $\equiv$ should be replaced by $\not\equiv$.
8. page 162, line 13, the correct formula is

$$\xi - \xi_0 = \ln\left[\left(\frac{T}{T_0}\right)\right] / \left(\frac{p_0}{p}\right)^{R/c_p}\Bigg]. \tag{8.1}$$

9. page 163, line 8, the correct expression is

$$\xi_0 = -\ln[(T_s / T_0)(p_o / p_s)^{R/c_p}].$$

10. page 163, line 12, the correct expression is

$$T(0,\lambda,\xi,t) = T_0\left\{\frac{p(0,\lambda,\xi,t)}{p_0}\right\}^{-R/c_p} e^{(\xi-\xi_0)} \tag{8.15}$$

11. page 167, lines 6 and 7, $\partial^2 Q(q^*,\xi) / \partial q^2$ should be replaced by $\partial^2 Q(q^*,\xi) / \partial q^2 < 0$.
12. page 170, line 18, $-r_0$ should be replaced by r_0.

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