Electronic Supplementary Material to: A Nonspherical Cloud Scattering Database Using Aggregates of Roughened Bullet Rosettes Model for the Advanced Radiative Transfer Modeling System (ARMS)*

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Introduction

The supplementary material includes the specific formulae for the ice crystal complex refractive index model developed by Mätzler, the simplification process of particle size distribution and the derivation of the effective diameter, a detailed derivation of the spherical harmonic expansion, as well as the complete formulae for the delta-*M* technique.

Refractive index of ice

The real part ε_i' and the imaginary part ε_i'' of the complex refractive index are calculated using the following formulae:

$$\varepsilon_i' = 3.1884 + 9.1 \times 10^{-4}(T - 273); \quad 243 \leqslant T \leqslant 273\ , \tag{S1}$$

$$\varepsilon_i'' = \frac{\alpha}{v} + \beta v\ , \tag{S2}$$

where T is the temperature in Kelvin, and v is the frequency in GHz. For the imaginary part of the complex refractive index, adjustments based on temperature are required for the α and β. The term α is corrected by term $\theta = T_0/T$, T_0 = 300 K:

$$\alpha = (0.00504 + 0.0062\theta)\exp(-22.1\theta)\ , \tag{S3}$$

and β is computed from two parts:

$$\beta = \beta_M + \Delta\beta\ , \tag{S4}$$

where

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$$\beta_M = \frac{B_1}{T}\frac{\exp(b/T)}{\left(\exp\left(\frac{b}{T}\right)-1\right)^2} + B_2 v^2 , \tag{S5}$$

and the correction term $\Delta\beta$ is required only when $T > 200$ K:

$$\Delta\beta = \exp(-9.963 + 0.0372(T - 273.16)) . \tag{S6}$$

Particle size distribution

The modified gamma distribution was selected in this paper and its complete form can be expressed as:

$$n(D) = \frac{N_0\mu}{\Gamma(\lambda)}\left(\frac{D}{D_N}\right)^{\mu\lambda-1}\frac{1}{D_N}\exp\left[-\left(\frac{D}{D_N}\right)^{\mu}\right] , \tag{S7}$$

where μ and λ are shape parameters, N_0 is the slope intercept of the function, Γ is the Gamma function, D_N is the scaling parameter, and its inverse is the slope of the distribution. The Gamma function is:

$$\Gamma(z) = \int_0^\infty x^{z-1}\exp(x)\mathrm{d}x . \tag{S8}$$

The μ and λ are both set to 1 in this paper, and the simplified form of this distribution is:

$$n(D) = N_0\frac{1}{D_N}\exp\left(-\frac{1}{D_N}D\right) . \tag{S9}$$

The bulk scattering properties are parameterized as a function of effective particle size D_{eff}, which also refers to the mean diameter of the cloud layer:

$$D_{\text{eff}} = \frac{\int_0^\infty D^3 n(D)\mathrm{d}D}{\int_0^\infty D^2 n(D)\mathrm{d}D} . \tag{S10}$$

For the modified gamma distribution used in this study, the effective particle could be simplified to:

$$D_{\text{eff}} = D_N\frac{\Gamma(4)}{\Gamma(3)} . \tag{S11}$$

Expansion coefficients of phase matrix

The optical scattering matrix of the scattering particles consists of six independent scattering phase matrix elements including P_{11}, P_{12}, P_{22}, P_{33}, P_{34}, P_{44} as

$$P(\theta) = \begin{bmatrix} P_{11}(\theta) & P_{12}(\theta) & 0 & 0 \\ P_{12}(\theta) & P_{22}(\theta) & 0 & 0 \\ 0 & 0 & P_{33}(\theta) & P_{34}(\theta) \\ 0 & 0 & -P_{34}(\theta) & P_{44}(\theta) \end{bmatrix} , \tag{S12}$$

where the $P_{11}(\theta)$ is normalized by Eq. (S12). The normalized scattering phase matrix elements could be expanded by generalized spherical functions $F_{m,n}^s$, where m and n are integers:

$$P_{11}(\theta) = \sum_{s=0}^{\infty} a_1^s F_{0,0}^s(\cos\theta) , \tag{S13}$$

$$P_{22}(\theta) + P_{33}(\theta) = \sum_{s=2}^{\infty}(a_2^s + a_3^s)F_{2,2}^s(\cos\theta) , \tag{S14}$$

$$P_{22}(\theta) - P_{33}(\theta) = \sum_{s=2}^{\infty}(a_2^s - a_3^s)F_{2,-2}^s(\cos\theta) \ , \tag{S15}$$

$$P_{44}(\theta) = \sum_{s=0}^{\infty} a_4^s F_{0,0}^s(\cos\theta) \ , \tag{S16}$$

$$P_{12}(\theta) = \sum_{s=2}^{\infty} b_1^s F_{0,2}^s(\cos\theta) \ , \tag{S17}$$

$$P_{34}(\theta) = \sum_{s=2}^{\infty} b_2^s F_{0,2}^s(\cos\theta) \ , \tag{S18}$$

where s is the expanded items, depending on the required numerical precision; a_1^s, a_2^s, a_3^s, a_4^s, b_1^s and b_2^s are the expansion coefficients of $P_{11}(\theta)$, $P_{22}(\theta)$, $P_{33}(\theta)$, $P_{44}(\theta)$, $P_{12}(\theta)$ and $P_{34}(\theta)$, respectively. The relationship between the Wigner d function and the generalized spherical function is as follows:

$$d_{mn}^s(\theta) = i^{n-m} F_{mn}^s(\theta) \ . \tag{S19}$$

The expansion coefficients of the scattering phase matrix elements could be derived through the relationship between the Wigner d function and the generalized spherical function:

$$a_1^s = \left(s + \frac{1}{2}\right)\int_{-1}^{1} d_{0,0}^s(\cos\theta) P_{11}(\theta) \, \mathrm{d}(\cos\theta) \ , \tag{S20}$$

$$a_2^s + a_3^s = \left(s + \frac{1}{2}\right)\int_{-1}^{1} d_{2,2}^s(\cos\theta)\left[P_{22}(\theta) + P_{33}(\theta)\right] \mathrm{d}(\cos\theta) \ , \tag{S21}$$

$$a_2^s - a_3^s = \left(s + \frac{1}{2}\right)\int_{-1}^{1} d_{2,-2}^s(\cos\theta)\left[P_{22}(\theta) - P_{33}(\theta)\right] \mathrm{d}(\cos\theta) \, , \tag{S22}$$

$$a_4^s = \left(s + \frac{1}{2}\right)\int_{-1}^{1} d_{0,0}^s(\cos\theta) P_{44}(\theta) \, \mathrm{d}(\cos\theta) \ , \tag{S23}$$

$$b_1^s = \left(s + \frac{1}{2}\right)\int_{-1}^{1} d_{0,2}^s(\cos\theta) P_{12}(\theta) \, \mathrm{d}(\cos\theta) \ , \tag{S24}$$

$$b_2^s = \left(s + \frac{1}{2}\right)\int_{-1}^{1} d_{0,2}^s(\cos\theta) P_{34}(\theta) \, \mathrm{d}(\cos\theta) \ . \tag{S25}$$

The Wigner d function has the following recurrence relation:

$$d_{mn}^{s+1}(\theta) = \frac{(2s+1)\left[s(s+1)\cos\theta - mn\right] d_{mn}^s(\theta) - (s+1)\sqrt{s^2 - m^2}\sqrt{s^2 - n^2} d_{mn}^{s-1}(\theta)}{s\sqrt{(s+1)^2 - m^2}\sqrt{(s+1)^2 - n^2}} \ , \tag{S26}$$

where $s \geqslant s_{\min}$, $s_{\min} = \max(|m|, |n|)$, while $s < s_{\min}$, the Wigner d function is equal to 0. The initial values of this recurrence relation are:

$$d_{mn}^{s_{\min}}(\theta) = \xi_{mn} 2^{-s_{\min}} \left[\frac{(2s_{\min})!}{(|m-n|)!\,(|m+n|)!}\right]^{1/2} (1 - \cos\theta)^{\frac{|m-n|}{2}} (1 + \cos\theta)^{\frac{|m+n|}{2}} \ . \tag{S27}$$

In addition, the $d_{mn}^{s_{\min}-1}(\theta) = 0$, and the ξ_{mn} can be expressed as:

$$\xi_{mn} = \begin{cases} 1 & n \geqslant m \\ (-1)^{m-n} & n < m \end{cases} . \tag{S28}$$

Note that all expansion coefficients must be multiplied by the normalization coefficient $1/\alpha_1^0$ to ensure numerical precision.

The original phase function could be approximately expressed as:

$$P(\cos\theta) \approx f_{\rm fws} \times 2\delta(1-\cos\theta) + (1-f_{\rm fws}) P'(\cos\theta) , \tag{S29}$$

where $f_{\rm fws}$ is the fraction of forward-scattering, δ is the Dirac delta function, and $P'(\cos\theta)$ is the truncated phase function, which is normalized if the original phase function is normalized, and could be expressed in another form:

$$P'(\cos\theta) = \sum_{l=0}^{M-1} C_l' P_l(\cos\theta) . \tag{S30}$$

Then, multiply the expression by $P_k(\cos\theta)$ and integrate the result by $\mu = \cos\theta$ from -1 to 1, using the orthogonal relationship of the Legendre polynomials:

$$\frac{C_k}{2k+1} \approx \begin{cases} f_{\rm fws} + (1-f_{\rm fws}) \dfrac{C_k'}{2k+1} & k = 0,1,2,\cdots,M-1 \\ f_{\rm fws} & k = M, M+1, \cdots \end{cases} . \tag{S31}$$

The appropriate coefficient C_k' in Eq. (S31) for $k \leqslant M-1$ is strictly equal, but the equation for $k \geqslant M$ is an approximation of the phase function, which could cause a truncation error. Due to the accuracy of low-level expansion being more significant than high-level, the $f_{\rm fws}$ could be represented as $C_M/(2M+1)$. Therefore, the coefficients of the truncated phase function could be expressed as:

$$C_l' \approx \frac{[C_l - f_{\rm fws}(2l+1)]}{(1-f_{\rm fws})} = \left[C_l - \left(\frac{2l+1}{2M+1}\right) C_M\right] \Big/ \left(1 - \frac{C_M}{2M+1}\right) . \tag{S32}$$

In ARMS, the coefficients computed by setting $2M$ to 4, 6, 8, 16 are written in a look-up table in proper order for two/four-stream, six-stream, eight-stream, sixteen-stream approximation.

Typhoon cases

Detailed information on the 14 typhoons over the western Pacific from 2022 to 2023 selected for simulation experiments in this study is provided in Table S1, below. All fields of view within the rainbands of these typhoons were included for deviation statistical analysis.

Table S1. List of typhoon cases for simulation experiments.

Typhoon name	Year	Time	Area
Haikui	2023	0700 UTC 3 September	19°–28°N 115°–125°E
Saola	2023	0800 UTC 30 August	15°–25°N 115°–124°E
Lan	2023	0800 UTC 11 August	24°–32°N 139°–146°E
Khanun	2023	2100 UTC 4 August	22°–31°N 122°–131°E
Doksuri	2023	0900 UTC 27 July	17°–26°N 114°–123°E
Guchol	2023	2000 UTC 10 June	17°–26°N 127°–136°E
Mawar	2023	2000 UTC 25 May	9°–20°N 133°–146°E
Nalgae	2022	2100 UTC 1 November	9°–20°N 110°–120°E
Roke	2022	0700 UTC 30 September	25°–35°N 135°–150°E
Noru	2022	1000 UTC 26 September	11°–20°N 107°–121°E
Muifa	2022	0800 UTC 11 September	19°–30°N 119°–127°E
Hinnamnor	2022	0800 UTC 5 September	26°–37°N 120°–130°E
Tokage	2022	0700 UTC 24 August	30°–38°N 146°–154°E
Malakas	2022	0700 UTC 14 April	17°–29°N 134°–151°E